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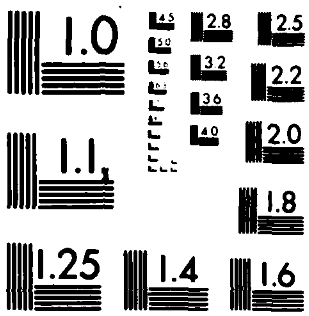
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ELECTROMAGNETIC COUPLING TO AN INFINITE WIRE THROUGH A SMALL APERTURE OF ARBITRARY SHAPE

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Electromagnetic coupling through an arbitrarily-shaped small aperture in an infinite conducting plane to an infinitely-long wire is investigated. The moment method is used to solve the integral equations numerically for the electric current on the wire and the equivalent magnetic current of the aperture. The general form of an equivalent circuit of the transmission line for the problem of a wire passing by an arbitrarily sized and shaped aperture is developed. Results for a small circular aperture are presented.

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CONTENTS

I. INTRODUCTION

The problem of electromagnetic coupling to a wire through a small aperture in an infinite conducting plane has been studied in the past. Kajfez [1] derived the equivalent sources and traveling wave current on the wire excited by a small aperture using both mode-matching and reciprocity, but he did not consider the interaction of the wire on the electromagnetic field in the aperture. For a small circular aperture whose diameter is much smaller than the shortest distance from the wire, Lee and Yang [2] obtained the same sources as those derived by Kajfez plus a lumped element equivalent circuit. The traveling current on the wire can be obtained from their equivalent circuit. When the restriction on the size of the aperture relative to the distance from the wire is removed, Lee and Yang modified the sources but not the lumped elements. However, the lumped circuit elements depend upon the interaction between the wire and the aperture, which should be taken into account when the wire gets closer to the aperture.

In this report, we obtain a pair of integral equations for the problem of a conducting wire passing by an aperture using the equivalence principle [3]. These equations are reduced to matrix form by means of the moment method [4]. We next specialize the equations to the problem of an arbitrarily-shaped small aperture and a thin infinitely-long wire. The electric current expansion functions on the wire are two outward traveling currents (TEM) on each semi-infinite half of the wire, plus pulse functions representing evanescent currents (higher-order modes) in a finite region near the aperture. The magnetic current expansion

functions on the aperture are dipoles located at the center of the aperture. For simplicity, a plane wave excitation from the region opposite the wire is assumed. Finally, an equivalent circuit for the TEM mode, including sources and lumped elements, is derived. We obtain both the outward traveling currents plus evanescent current on the wire, and the equivalent dipole current elements of the small aperture.

II. PROBLEM FORMULATION

The general problem configuration is given in Fig. 2.1, which shows an arbitrary aperture (labeled A) in an infinite conducting screen separating regions a and b. In region a ($y < 0$), the impressed sources $\underline{J}^{ia}$ and $\underline{M}^{ia}$ produce short-circuited fields (fields radiating in the presence of the complete screen) $\underline{E}^{ia}$ and $\underline{H}^{ia}$. In region b ($y > 0$), the impressed sources $\underline{J}^{ib}$ and $\underline{M}^{ib}$ produce short-circuited fields $\underline{E}^{ib}$ and $\underline{H}^{ib}$. There is also a conducting wire labeled B in region b. The loss-free homogeneous material is characterized by μ_a, ϵ_a in region a, and by μ_b, ϵ_b in region b. The problem is to find the currents on the wire and in the aperture, and the parameters of the equivalent network seen by the TEM traveling wave on the wire.

Following the development of [5], we treat the problem as a boundary value problem for which tangential components of both $\underline{E}$ and $\underline{H}$ are continuous across the aperture and the tangential component of $\underline{E}$ on the surface of the wire vanishes, and then use the equivalence principle [3]. In region b, the field is expressed in terms of the equivalent magnetic current $\underline{M} = \underline{n} \times \underline{E}$ over the aperture and the current $\underline{J}$ on the wire. Here $\underline{M}$ corresponds to $-\underline{M}$ in [5], $\underline{n}$ is the unit vector normal to A and pointing to B, and $\underline{E}$ is the electric field in A of the original problem.

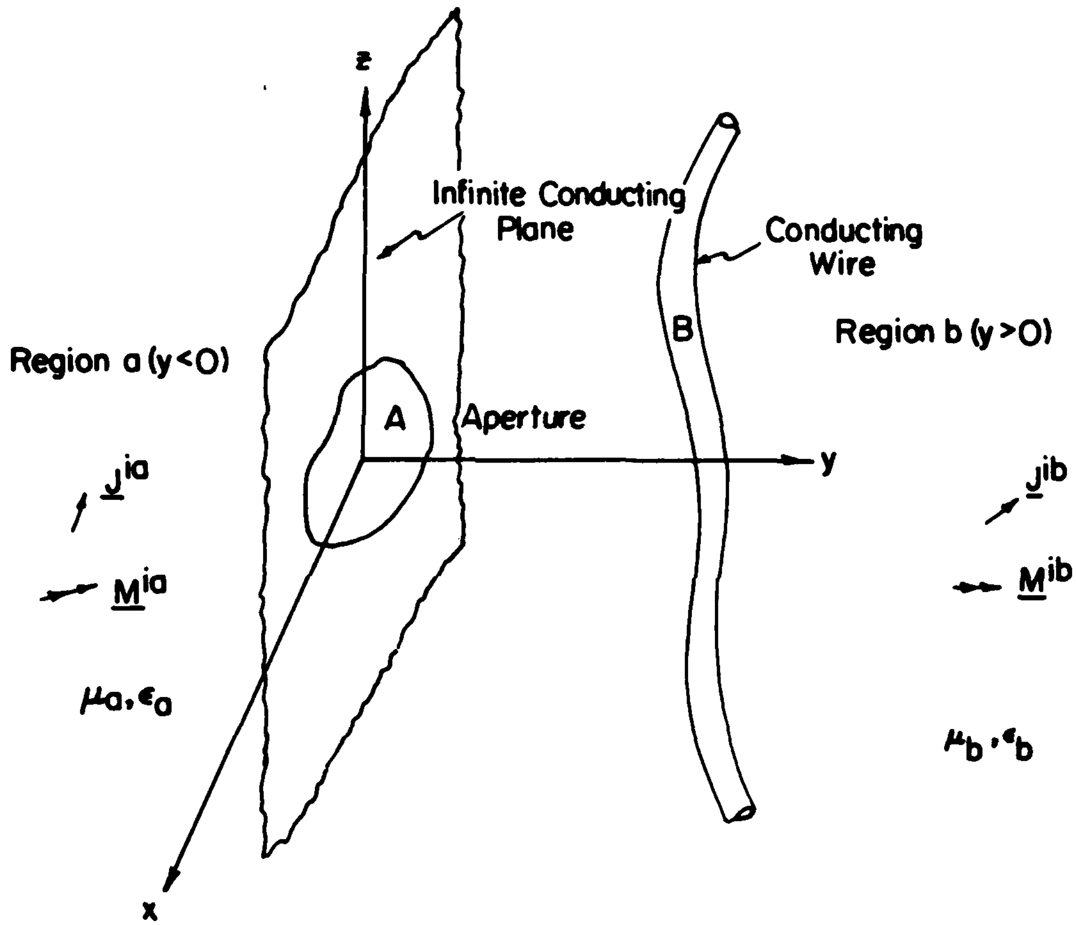


Fig. 2.1. The general problem.

The field in region a is that due to $-\underline{M}$ plus the impressed field. To compute $\underline{M}$ and $\underline{J}$, we use method of moments [4] and expand $\underline{M}$ and $\underline{J}$ as $\underline{M} = \sum_{n=1}^{NA} V_n \underline{M}_n$ and $\underline{J} = \sum_{n=1}^{NB} I_n \underline{J}_n$, where NA and NB are the total number of expansion functions over A and B, respectively, and V_n, I_n are the coefficients to be evaluated. A pair of matrix equations are then obtained,

$$[Y^a + Y^b]\vec{V} + [T]\vec{I} = \vec{I}^i \tag{2-1}$$

$$[\hat{T}]V + [Z]I = \vec{V}^i \tag{2-2}$$

where

$$[Y^{\substack{a\\b}}] = [\langle -\hat{\underline{M}}_m, \underline{H}_t^{\substack{a\\b}}(\underline{M}_n)\rangle_A]_{NA \times NA} \tag{2-3}$$

$$[T] = [\langle -\hat{\underline{M}}_m, \underline{H}_t^{b}(\underline{J}_n)\rangle_A]_{NA \times NB} \tag{2-4}$$

$$[\hat{T}] = [\langle -\hat{\underline{J}}_m, \underline{E}_t^{b}(\underline{M}_n)\rangle_B]_{NB \times NA} \tag{2-5}$$

$$[Z] = [\langle -\hat{\underline{J}}_m, \underline{E}_t^{b}(\underline{J}_n)\rangle_B]_{NB \times NB} \tag{2-6}$$

$$\vec{I}^i = [\langle \hat{\underline{M}}_m, \underline{H}_t^{ib} - \underline{H}_t^{ia}\rangle_A]_{NA \times 1} \tag{2-7}$$

$$\vec{V}^i = [\langle \underline{J}_m, \underline{E}_t^{ib}\rangle_B]_{NB \times 1} \tag{2-8}$$

$$\vec{V} = [V_n]_{NA \times 1} \tag{2-9}$$

$$\vec{I} = [I_n]_{NB \times 1} \tag{2-10}$$

Here, $\hat{\underline{M}}_m$ and $\hat{\underline{J}}_m$ are the testing functions for $\underline{M}$ and $\underline{J}$, respectively. Subscripts A and B denote symmetric products over A and B, respectively. Subscript t denotes the component tangential to the surface (A or B) where the product is applied. Superscript a denotes region a, and

superscript b denotes region b. $\{\underline{H}(\underline{M}_n), \underline{E}(\underline{M}_n)\}$ is the field due to $\underline{M}_n$, and $\{\underline{H}(\underline{J}_n), \underline{E}(\underline{J}_n)\}$ is the field due to $\underline{J}_n$. Note that all fields are computed with the aperture shorted.

If a Galerkin solution is used (i.e. $\hat{\underline{M}}_m = \underline{M}_m$, $\hat{\underline{J}}_m = \underline{J}_m$), and regions a and b of Fig. 2-1 are filled with the same material, we can use reciprocity [3, sec. 3-8] and image theory [3, sec. 3-4] to reduce (2-3) - (2-8) to

$$Y^a_{mn} = Y^b_{mn} = Y^a_{nm} = 2\langle -\underline{M}_m, \underline{H}_t(\underline{M}_n)\rangle_A \tag{2-11}$$

$$T_{mn} = \langle -\underline{M}_m, \underline{H}_t(\underline{J}_n, \underline{J}'_n)\rangle_A \tag{2-12}$$

$$\hat{T}_{mn} = 2\langle -\underline{J}_m, \underline{E}_t(\underline{M}_n)\rangle_B = -T_{nm} \tag{2-13}$$

$$Z_{mn} = Z_{nm} = \langle -\underline{J}_m, \underline{E}_t(\underline{J}_n, \underline{J}'_n)\rangle_B \tag{2-14}$$

$$I^i_m = 2\langle \underline{M}_m, \underline{H}^{iob}_t - \underline{H}^{ioa}_t\rangle_A \tag{2-15}$$

$$V^i_m = 2\langle \underline{J}_m, \underline{E}^{iob}_t\rangle_B \tag{2-16}$$

where $\underline{J}'_n$ is the image of $\underline{J}_n$ and located in region a. All the fields are evaluated with the conducting plane removed.

III. PROBLEM SPECIALIZATION

In this section, we specialize the problem to a thin infinitely-long conducting wire of radius r_B passing by a small arbitrarily-shaped aperture centered at the coordinate origin. Space is filled with loss-free homogeneous material of permeability μ and permittivity ε on both sides of the screen. The impressed field in region a is assumed to be

a plane wave. For simplicity, we consider horizontal polarization for the incident plane wave. The incident fields with the screen removed are

$$\underline{H}^{ioa} = -\underline{u}_x \frac{e^{-jkz}}{\eta} \tag{3-1}$$

$$\underline{E}^{ioa} = \underline{u}_y e^{-jkz} \tag{3-2}$$

where $\underline{u}_x$ and $\underline{u}_y$ are the unit vectors in the x and the y directions, respectively. $\eta = \sqrt{\mu/\epsilon}$ denotes the intrinsic impedance of free space, and $k = 2\pi/\lambda$ where λ is the wavelength. There is no incident wave from region b. The configuration of the specified problem is shown in Fig. 3.1.

A Galerkin solution is used to solve the problem, and we specify the magnetic current expansion functions in the aperture and electric current expansion functions on the wire as follows.

(A) Magnetic Current Expansions in the Small Aperture

As described in [6], the field radiated by a small aperture is the combination of that from a magnetic dipole $\underline{p}_m$ tangential to the screen, and an electric dipole $\underline{p}_e$ normal to the screen, both located at the center of the aperture [7], [8], [9]. The magnetic current $\underline{M}$ on the aperture can then be expressed by a three-term expansion,

$$\underline{M} = \sum_{n=1}^{3} V_n \underline{M}_n \tag{3-3}$$

where $\underline{M}_1$ and $\underline{M}_2$ are quasi-static magnetic currents which produces the effects of unit magnetic dipoles $\underline{K\ell} = \underline{t}_1$ and $\underline{K\ell} = \underline{t}_2$, respectively. Here $\underline{t}_1$ and $\underline{t}_2$ are unit vectors tangential to the screen chosen to

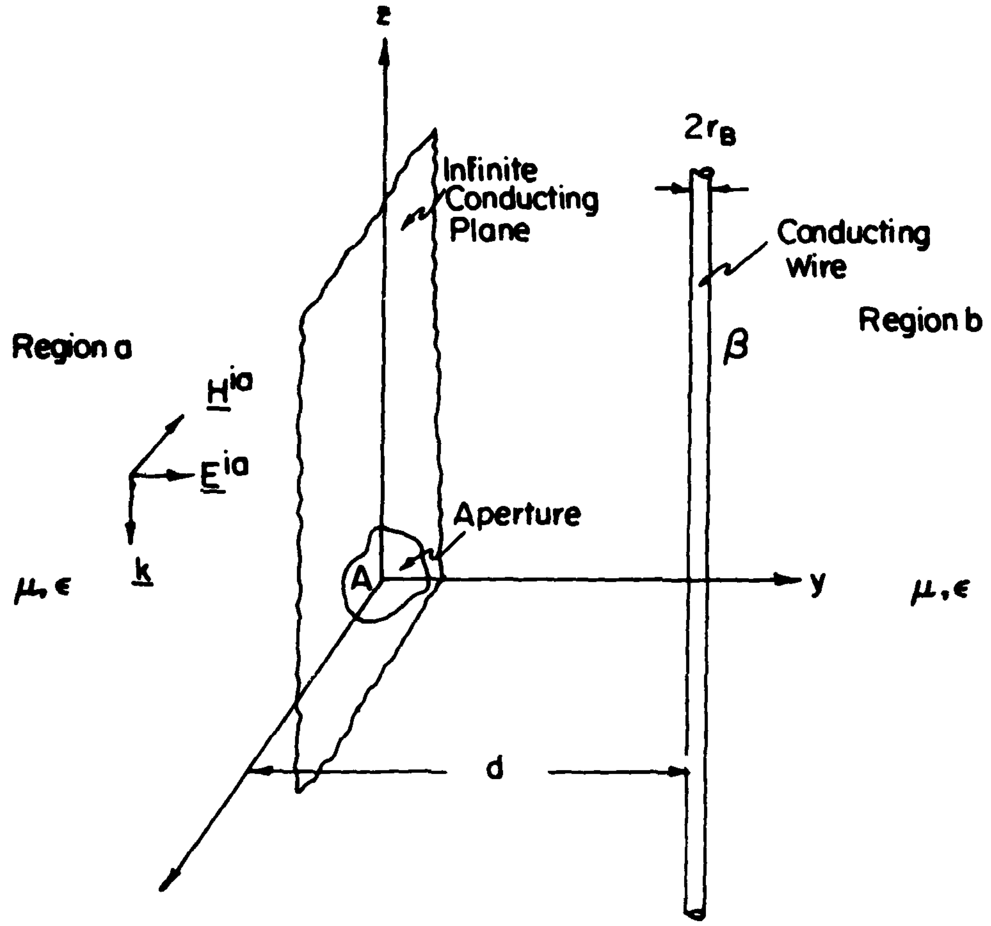


Fig. 3.1. A thin infinitely-long wire passing by a small aperture.

diagonalize the magnetic polarizability $\bar{\alpha}_m$. For simplicity they are assumed in the x and the z directions. $\underline{M}_3$ is a quasi-static electric current producing the effect of an electric dipole $\underline{I\ell} = -j\omega\,\underline{n}$. Here $\underline{n}$ is a unit vector normal to the screen and is chosen in the y direction. These dipoles are located at center $\underline{0}$ of the aperture.

(B) Electric Current Expansions on the Wire

We make the following approximations for the current on a thin wire: (a) The current flows only in the axial direction of the wire, and (b) depends only on the axial length variable. We next note that the wire and the complete screen form a transmission line. We therefore expect that the aperture excites two outward traveling (TEM) waves propagating in the -z and the +z directions without attenuation, plus evanescent waves existing near the aperture. It is convenient to divide a section of the wire near the aperture into NB-2 equally-spaced subsections of length $2\Delta_B$. Pulse functions over each subsection are used. The expansion functions for the current $\underline{J}$ on the wire are then

$$\underline{J}_1 = \underline{u}_z\, e^{jkz} \quad , \qquad -\infty < z < 0 \tag{3-4}$$

$$\underline{J}_{NB} = \underline{u}_z\, e^{-jkz} \quad , \qquad 0 < z < \infty \tag{3-5}$$

$$\underline{J}_n = \underline{u}_z\, P(z-z_n) \quad , \qquad n = 2,3,\ldots, NB-1 \tag{3-6}$$

$$|z| \leq \frac{WL}{2}$$

$$P(z-z_n) = \begin{cases} 1 & \text{for} \quad z_n^- \leq z \leq z_n^+ \\ 0 & \text{elsewhere} \end{cases} \tag{3-7}$$

where WL is the length of the section over which the evanescent current is assumed to exist. $z_{\bar{n}}$, z_n and $z_{\overset{+}{n}}$ are the z-coordinates of the starting point $\bar{n}$, midpoint n and termination $\overset{+}{n}$ of the nth subsection, and the numbering starts at $z = -\frac{WL}{2}$.

IV. EVALUATION OF MATRICES

In this section we evaluate the matrices defined in (2-11) - (2-16) for the problem of Fig. 3.1.

(A) Impedance Matrix [Z]

The tangential component of electric field on the surface of the wire due to current expansion $\underline{J}_n$ at the rectangular coordinate (0, d, z') and its image $\underline{J}'_n = -\underline{J}_n$ at (0, -d, z') are expressed in terms of potential integrals as [4, Eqs. (4-6) - (4-9)]

$$E_z(\underline{J}_n, \underline{J}'_n) = -\frac{j\omega\mu}{4\pi}\int_B J_n(z')\left(\frac{e^{-jkr}}{r} - \frac{e^{-jkr'}}{r'}\right)dz'$$

$$+\frac{1}{4\pi j\omega\epsilon}\frac{\partial}{\partial z}\int_B \frac{dJ_n(z')}{dz'}\left(\frac{e^{-jkr}}{r} - \frac{e^{-jkr'}}{r'}\right)dz' \qquad (4\text{-}1)$$

$$, \; n = 1, 2, 3, \ldots, NB$$

Here

$$r = \sqrt{(z - z')^2 + r_B^2} \qquad (4\text{-}2)$$

is the distance between a field point on the surface of the wire and a source point of $J_n(z')$ on the axis of the wire, and

$$r' = \sqrt{(z - z')^2 + (2d)^2} \qquad (4\text{-}3)$$

is the distance between the field point and the image of the source point.

We now evaluate the elements of [Z] matrix defined in (2-14). For $m \neq 1$, NB, and $n \neq 1$, NB, it is the [Z] matrix of a wire of length $wL + 2\,\alpha_B$ above a ground plane. We use pulse functions for both $J_n(z')$ and $\frac{\partial J_n(z')}{\partial z'}$ in (4-1) and point matching for testing in (2-14), and extend the derivations of [4, chap. 4] to include the $\underline{E}$ due to the image of the wire. The results are

$$
\begin{aligned}
Z_{mn} = {} & j\omega\mu\, \Delta\ell_m \Delta\ell_n [\psi(z_m, z_n, r_B) - \psi(z_m, z_n, 2d)] \\
& + \frac{1}{j\omega\epsilon} [\psi(z_{m}^{+}, z_{n}^{+}, r_B) - \psi(z_{m}^{+}, z_{n}^{+}, 2d) \\
& - \psi(z_{m}^{+}, z_{n}^{-}, r_B) + \psi(z_{m}^{+}, z_{n}^{-}, 2d) \\
& - \psi(z_{m}^{-}, z_{n}^{+}, r_B) + \psi(z_{m}^{-}, z_{n}^{+}, 2d) \\
& + \psi(z_{m}^{-}, z_{n}^{-}, r_B) - \psi(z_{m}^{-}, z_{n}^{-}, 2d)]
\end{aligned}
\tag{4-4}
$$

where

$$
\psi(z_m, z_n, \rho) = \frac{1}{4\pi\Delta\ell_n} \int_{\Delta\ell_n} \frac{e^{-jk\sqrt{(z_m - z')^2 + \rho^2}}}{\sqrt{(z_m - z')^2 + \rho^2}}\, dz' \tag{4-5}
$$

has been evaluated in [10].

For $m = 1, 2, \ldots,$ NB and $n = 1$, NB, from (2-14) we have

$$
Z_{m,1} = -\langle J_m, E_z(\underline{J}_1, \underline{J}_1') \rangle_B \tag{4-6}
$$

$$
Z_{m,NB} = -\langle J_m, E_z(\underline{J}_{NB}, \underline{J}_{NB}') \rangle_B \tag{4-7}
$$

By [11, Eqs. (A-9), (A-12)] and (4-1), we have

$$E_z(\underline{J}_1, \underline{J}_1') = \frac{j}{4\pi\omega\epsilon} \left[\left(\frac{jk}{r_o} - \frac{jkz}{r_o^2} - \frac{z}{r_o^3} \right) e^{-jkr_o} \right.$$

$$- \left(\frac{jk}{r_o'} - \frac{jkz}{r_o'^2} - \frac{z}{r_o'^3} \right) e^{-jkr_o'} \tag{4-8}$$

$$E_z(\underline{J}_{NB}, \underline{J}_{NB}') = \frac{j}{4\pi\omega\epsilon} \left[\left(\frac{jk}{r_o} + \frac{jkz}{r_o^2} + \frac{z}{r_o^3} \right) e^{-jkr_o} \right.$$

$$\left. - \left(\frac{jk}{r_o'} + \frac{jkz}{r_o'^2} + \frac{z}{r_o'^3} \right) e^{-jkr_o'} \right] \tag{4-9}$$

where

$$r_o = \sqrt{z^2 + r_B^2} \tag{4-10}$$

$$r_o' = \sqrt{z^2 + (2d)^2} \tag{4-11}$$

However, we note that the charge densities of $\underline{J}_1$ and $\underline{J}_{NB}$ are

$$\frac{dJ_1}{dz} = jk\, e^{jkz} - \delta(z) \qquad -\infty < z < 0 \tag{4-12}$$

$$\frac{dJ_{NB}}{dz} = -jk\, e^{-jkz} + \delta(z) \quad 0 < z < \infty \tag{4-13}$$

and $\underline{E}_z$ due to these point charges $-\delta(0^-)$ and $\delta(0^+)$ are exactly the terms involving r_o^{-2}, r_o^{-3}, $r_o'^{-2}$ and $r_o'^{-3}$ in (4-8) and (4-9). In order to assure the continuity of the total current (outward traveling current plus evanescent current) on the wire, we first replace $\delta(z)$ in (4-12) and (4-13) by a pulse function $\frac{P(z)}{\Delta z_0}$ over a small region Δz_0 centered at $z = 0$.

Here $\Delta\ell_0$ is chosen, for simplicity, as the same length as each evanescent-current-subsection. We next replace the $\underline{E}$ due to charge density $\delta(z)$ by the $\underline{E}$ due to charge density $\frac{P(z)}{\Delta\ell_0}$. Thus (4-8) and (4-9) become

$$E_z(\underline{J}_1, \underline{J}_1') = \frac{-k}{4\pi\omega\epsilon} \left(\frac{e^{-jkr_o}}{r_o} - \frac{e^{-jkr_o'}}{r_o'}\right) - \frac{1}{4\pi j\omega\epsilon\,\Delta\ell_0} \frac{\partial}{\partial z} \int_{\Delta\ell_0} \left(\frac{e^{-jkr}}{r} - \frac{e^{-jkr'}}{r'}\right) dz' \qquad (4\text{-}14)$$

$$E_z(\underline{J}_{NB}, \underline{J}_{NB}') = \frac{-k}{4\pi\omega\epsilon} \left(\frac{e^{-jkr_o}}{r_o} - \frac{e^{-jkr_o'}}{r_o'}\right) + \frac{1}{4\pi j\omega\epsilon\,\Delta\ell_0} \frac{\partial}{\partial z} \int_{\Delta\ell_0} \left(\frac{e^{-jkr}}{r} - \frac{e^{-jkr'}}{r'}\right) dz' \qquad (4\text{-}15)$$

where r, r', r_o and r_o' are defined in (4-2), (4-3), (4-10) and (4-11), respectively.

Substituting (4-14) into (4-6), and using the integration formula [11, Eq. (37)], we obtain

$$Z_{11} = \frac{1}{j\omega\epsilon} [\psi(0, 0, r_B) - \psi(0, 0, 2d)] \qquad (4\text{-}16)$$

$$Z_{NB,1} = \frac{k}{2\pi\omega\epsilon} \{- C_i(r_B k) + C_i(2dk) + j [S_i(r_B k) - S_i(2dk)]\} - \frac{1}{j\omega\epsilon} [\psi(0, 0, r_B) - \psi(0, 0, 2d)] \qquad (4\text{-}17)$$

$$Z_{m1} = \frac{k\,\Delta\ell_m}{[illegible]} [\psi(Z_m, 0, r_B) - \psi(z_m, 0, 2d)]$$

(continued on next page)

$$+ \frac{1}{j\omega\varepsilon} [\psi(z_{m_+}, 0, r_B) - \psi(z_{m_+}, 0, 2d)$$

$$- \psi(z_{m_-}, 0, r_B) + \psi(z_{m_-}, 0, 2d)] \tag{4-18}$$

for $m = 2, 3, \ldots, NB-1$.

Here ψ is defined in (4-5), and C_i and S_i are cosine and sine integrals, respectively.

Substituting (4-15) into (4-7), and using [11, Eq. (37)], we obtain

$$Z_{NB,NB} = Z_{11} \tag{4-19}$$

$$Z_{M,NB} = \frac{k\,\Delta\ell_m}{\omega\varepsilon} [\psi(z_m, 0, r_B) - \psi(z_m, 0, 2d)]$$

$$- \frac{1}{j\omega\varepsilon} [\psi(z_{m_+}, 0, r_B) - \psi(z_{m_+}, 0, 2d) \tag{4-20}$$

$$- \psi(z_{m_-}, 0, r_B) + \psi(z_{m_-}, 0, 2d)]$$

for $m = 2, 3, \ldots, NB-1$.

Since a Galerkin solution is used, the rest of the impedance elements can be obtained using $Z_{mn} = Z_{nm}$ for $m, n = 1,2,\ldots,NB$.

<u>(B) Source Vectors $\vec{V}^i$, $\vec{I}^i$ and Admittance Matrix $[Y^a + Y^b]$</u>

Since $\underline{E}^{ib} = \underline{0}$ in (2-16), it is obvious that

$$\vec{V}^i = \vec{0} \tag{4-21}$$

As developed in [6], the source vector $\vec{I}^i$ and admittance matrix $[Y^a + Y^b]$ for a small aperture in Fig. 2.1 can be obtained as follows. We first interpret $\underline{M}_3$ as an electric dipole $\underline{I\ell} = j\omega\varepsilon\,\underline{n}_a$ if it is in

region a and $I_\ell^c = -j\omega\epsilon_b \underline{n}$ if it is in region b, while $\underline{M}_1$ and $\underline{M}_2$ are the same as those defined in Section III. Applications of reciprocity [3, Sec. 3-8] to (2-7) and the use of the definitions of $\underline{M}_1$, $\underline{M}_2$ and $\underline{M}_3$ give the source vector $\vec{I}^i$

$$I_1^i = H_1^{ib}(\underline{0}) - H_1^{ia}(\underline{0}) \tag{4-22}$$

$$I_2^i = H_2^{ib}(\underline{0}) - H_2^{ia}(\underline{0}) \tag{4-23}$$

$$I_3^i = j\omega\,[\epsilon_b E_3^{ib}(\underline{0}) - \epsilon_a E_3^{ia}(\underline{0})] \tag{4-24}$$

Here, H_1 and H_2 denote the $\underline{t}_1$ and the $\underline{t}_2$ components of $\underline{H}$, and E_3 denotes the $\underline{n}$ component of $\underline{E}$. All the fields are evaluated at the center $\underline{0}$ of the aperture.

The imaginary part of $[Y^a + Y^b]$ can be evaluated by the extension of the Bethe-hole theory [7] to allow for contrasting media [12]. Thus, the field due to $-\underline{M}$ in region a of Fig. 2.1 can be approximated by the combination of that of a magnetic dipole with current element $-\underline{K}\ell$ plus that of an electric dipole with current element $-\underline{I}\ell$. The field due to $\underline{M}$ in region b of Fig. 2.1 can be approximated by the combination of that of a magnetic dipole with current element $\underline{K}\ell' = \underline{K}\ell$ plus that of an electric dipole with current element $\underline{I}\ell' = \frac{\epsilon_b}{\epsilon_a}\underline{I}\ell$. In addition, the relationships among dipole moments, incident fields and these current elements are

$$\underline{K}\ell = j\omega\mu_a \underline{p}_m \tag{4-25}$$

$$\underline{I}\ell = j\omega \underline{p}_e \tag{4-26}$$

$$\underline{P}_m = \frac{2\mu_b}{\mu_a + \mu_b} \{- \underline{t}_1 \alpha_{m1} [H_1^{ia}(\underline{0}) - H_1^{ib}(\underline{0})]$$

$$- \underline{t}_2 \alpha_{m2} [H_2^{ia}(\underline{0}) - H_2^{ib}(\underline{0})]\} \quad (4\text{-}27)$$

$$\underline{P}_e = \frac{2\varepsilon_a}{\varepsilon_a + \varepsilon_b} \underline{n} \alpha_e [\varepsilon_a E_3^{ia}(\underline{0}) - \varepsilon_b E_3^{ib}(\underline{0})] \quad (4\text{-}28)$$

Here α_{m1} and α_{m2} are the $\underline{t}_1$ and $\underline{t}_2$ components of magnetic polarizability, and α_e the electric polarizability. The relationships between the coefficient vector $\vec{V}$ of $\underline{M}$ and these current elements $\underline{K\ell}$, $\underline{I\ell}$ can be obtained from the definitions of $\underline{M}_1$, $\underline{M}_2$ and $\underline{M}_3$. They are

$$\underline{K\ell} = V_1 \underline{t}_1 + V_2 \underline{t}_2 \quad (4\text{-}29)$$

$$\underline{I\ell} = -j\omega\varepsilon_a V_3 \underline{n} \quad (4\text{-}30)$$

By (2-1) with $\vec{I} = \vec{0}$ ($[Y^a + Y^b]$ can be obtained without the presence of the wire) and (4-22) - (4-30), we have

$$[Y^a + Y^b] = \begin{bmatrix} \frac{\mu_a + \mu_b}{2j\omega\mu_a\mu_b\alpha_{m1}} & 0 & 0 \\ 0 & \frac{\mu_a + \mu_b}{2j\omega\mu_a\mu_b\alpha_{m2}} & 0 \\ 0 & 0 & \frac{j\omega(\varepsilon_a + \varepsilon_b)}{2\alpha_e} \end{bmatrix} \quad (4\text{-}31)$$

The admittance matrix in (4-31) came out purely imaginary because no radiation terms were included in the equivalent dipoles of the small aperture in (4-27) and (4-28), that is, only the first term of a frequency expansion for $[Y^a + Y^b]$ is evaluated. When there is an object

near the aperture, as in our original problem, the first term of expansion for $[Y^a + Y^b]$ can be cancelled by the interaction between the object and the aperture. Then the second term of frequency expansion for $[Y^a + Y^b]$, which is its real part, becomes important.

The real part of $[Y^a + Y^b]$ can be obtained from the power radiated by $\underline{M}$. Note that $\underline{M}_n$ are real magnetic currents ($\underline{M}_3$ equivalent to a magnetic current loop $KS = \frac{1}{\omega\mu_b}$ [3, p. 135]), and for $\underline{M}_n$ real, we have $\mathrm{Re}(Y^a_{nn} + Y^b_{nn})$ equal to the power radiated by $\underline{M}_n$ in regions a and b. Using the formula for the power radiated by an electric dipole [3, Eq. (2-116)], duality [3, sec. 3-2] and definitions of $\underline{M}_n$, we obtain

$$\mathrm{Re}(Y^a_{11}) + \mathrm{Re}(Y^b_{11}) = \frac{1}{3\pi}\left(\frac{k_a^2}{\eta_a} + \frac{k_b^2}{\eta_b}\right) \tag{4-32}$$

$$\mathrm{Re}(Y^a_{22}) + \mathrm{Re}(Y^b_{22}) = \frac{1}{3\pi}\left(\frac{k_a^2}{\eta_a} + \frac{k_b^2}{\eta_b}\right) \tag{4-33}$$

$$\mathrm{Re}(Y^a_{33}) + \mathrm{Re}(Y^b_{33}) = \frac{1}{3\pi}\left(\frac{k_a^4}{\eta_a} + \frac{k_b^4}{\eta_b}\right) \tag{4-34}$$

where $\eta_a = \sqrt{\frac{\epsilon_a}{\mu_a}}$, $k_a = \frac{2\pi}{\lambda_a}$, $k_b = \frac{2\pi}{\lambda_b}$ and $\eta_b = \sqrt{\frac{\epsilon_b}{\mu_b}}$. Combining (4-31) - (4-34) we obtain the admittance matrix for a small aperture in Fig. 2.1.

For the problem of Fig. 3.1, we have $\epsilon_a = \epsilon_b = \epsilon$, $\lambda_a = \lambda_b = \lambda$ and $\mu_a = \mu_b = \mu$. Thus, the source vector I^i can be obtained by (4-22)-(4-24) with $\underline{H}^{ib} = 0$, $\underline{E}^{ib} = \underline{0}$ and $\underline{H}^{ia}$, $\underline{E}^{ia}$ defined in (3-1) and (3-2). The result is

$$\begin{bmatrix} I_1^i \\ I_2^i \\ I_3^i \end{bmatrix} = \begin{bmatrix} \frac{2}{\eta} \\ 0 \\ -2j\omega\epsilon \end{bmatrix} \tag{4-35}$$

The admittance matrix of (4-31)-(4-34) can be reduced to

$$[Y^a + Y^b] = \begin{bmatrix} \dfrac{8\pi}{3\eta\lambda^2} + \dfrac{1}{j\omega\mu\,\alpha_{m1}} & 0 & 0 \\ 0 & \dfrac{8\pi}{3\eta\lambda^2} + \dfrac{1}{j\omega\mu\,\alpha_{m2}} & 0 \\ 0 & 0 & \dfrac{32\pi^3}{3\eta\lambda^4} + \dfrac{j\omega}{\alpha_e} \end{bmatrix} \tag{4-36}$$

(C) Coupling Matrices [T] and $[\tilde{T}]$

Since a Galerkin solution is used, $[\tilde{T}]$ can be obtained in terms of [T] according to (2-13). Therefore, we only have to evaluate [T].

Application of reciprocity to the right-hand side of (2-12) gives

$$T_{mn} = \iint_B \underline{J}_n \cdot \underline{E}(2\underline{M}_n)\,ds \tag{4-37}$$

Once again, application of reciprocity to the right-hand side of (4-37) and use of the impulsive nature of $\underline{M}_n$ give

$$T_{mn} = \begin{cases} -H_m(\underline{J}_n, \underline{J}_n' ; \underline{0}) & , \quad m = 1,2 \\ -j\omega\, E_3(\underline{J}_n, \underline{J}_n' ; \underline{0}) & , \quad m = 3 \end{cases}$$

$$\text{for } n = 1,2,\ldots, NB \tag{4-38}$$

Here $\underline{H}_m(\underline{J}_n, \underline{J}_n'; \underline{0})$ and $E_3(\underline{J}_n, \underline{J}_n'; \underline{0})$ are the fields due to $\underline{J}_n$ and its image $\underline{J}_n'$ evaluated at $\underline{0}$.

The x and the z components of $\underline{H}$ fields due to $\underline{J}_1$ and $\underline{J}_{NB}$ at the center $\underline{0}$ of the aperture can be obtained by [11, Eqs. (A-6) and (A-11)]

with the coordinate of $\underline{0}$ at $\rho = d$, $\phi = \frac{3\pi}{2}$ and $z = 0$ in the geometry system of [11, Fig. A-1].

$$H_x(\underline{J}_1\ ;\ \underline{0}) = H_x(\underline{J}_{NB}\ ;\ \underline{0})$$

$$= -\sin\phi\ H\ (\underline{J}_1\ ;\ \underline{0})$$

$$= \frac{e^{-jkd}}{4\pi d} \tag{4-39}$$

$$H_z(\underline{J}_1\ ;\ \underline{0}) = H_z(\underline{J}_{NB}\ ;\ \underline{0})$$

$$= 0 \tag{4-40}$$

The y component of $\underline{E}$ fields due to $\underline{J}_1$ can be obtained by using [11, Eq. (A-11)], giving

$$E_y(\underline{J}_1) = \sin\phi\ E_\rho\ (\underline{J}_1)$$

$$= \frac{\sin\phi}{j\omega\epsilon}\ \frac{\partial}{\partial z}\ H_\phi(\underline{J}_1)$$

$$= \frac{\sin\phi}{4\pi j\omega\epsilon}\ [\frac{1}{r_o} + \frac{jkz}{r_o} - \frac{jkz^2}{r_o} - \frac{z^2}{r_o^3}]\ e^{-jkr_o} \tag{4-41}$$

where $r_o = \sqrt{z^2 + \rho^2}$. Again, at $\underline{0}$ we have $\rho = d$, $\phi = \frac{3\pi}{2}$ and $z = 0$, and (4-41) becomes

$$E_y(\underline{J}_1\ ;\ \underline{0}) = \frac{-e^{-jkd}}{4\pi j\omega\epsilon\ d^2} \tag{4-42}$$

Similarly, using [11, Eq. (A-6)], we have

$$E_y(\underline{J}_{NB}\ ;\ \underline{0}) = -\ E_y(\underline{J}_1\ ;\ \underline{0}) \tag{4-43}$$

The ϕ-component of $\underline{H}$ due to $\underline{J}_n$ for $n = 2,3,\ldots,NB-1$ are

$$H_\phi(\underline{J}_n) = -\frac{\partial A_z(\underline{J}_n)}{\partial \rho}$$

$$= -\frac{1}{4\pi}\frac{\partial}{\partial \rho}\int_{\Delta_n} \frac{e^{-jk\sqrt{(z-z')^2+\rho^2}}}{\sqrt{(z-z')^2+\rho^2}}\,dz'$$

$$\approx \frac{\Delta_n \rho}{4\pi}\left(\frac{1}{r_n^3}+\frac{jk}{r_n^2}\right)e^{-jkr_n} \tag{4-44}$$

where $r_n = \sqrt{(z-z_n)^2+\rho^2}$. Thus, the x and the z components of $\underline{H}(\underline{J}_n)$ evaluated at $\underline{0}$ are

$$H_x(\underline{J}_n;\underline{0}) = -\sin\phi\, H_\phi(\underline{J}_1;\underline{0})$$

$$= \frac{\Delta_n d}{4\pi}\left(\frac{1}{r_{on}^3}+\frac{jk}{r_{on}^2}\right)e^{-jkr_{on}} \tag{4-45}$$

$$H_z(\underline{J}_n;\underline{0}) = 0 \tag{4-46}$$

where

$$r_{on} = \sqrt{z_n^2+d^2} \tag{4-47}$$

The y component of $\underline{E}$ due to $\underline{J}_n$ at $\underline{0}$ can be obtained by using $E_y = \frac{\sin\phi}{j\omega\epsilon}\frac{\partial}{\partial z}H_\phi$ evaluated at $\underline{0}$, giving

$$E_y(\underline{J}_n;\underline{0}) = \frac{-d}{4\pi j\omega\epsilon}\Big[\left(\frac{1}{r_{n^+}^3}+\frac{jk}{r_{n^+}^2}\right)e^{-jkr_{n^+}}$$

$$-\left(\frac{1}{r_{n^-}^3}+\frac{jk}{r_{n^-}^2}\right)e^{-jkr_{n^-}}\Big] \tag{4-48}$$

for n = 2,3,...., NB-1.

where

$$r_{\overset{+}{n}} = \sqrt{z_{\overset{+}{n}}^2 + d^2} \tag{4-49}$$

$$r_{\overset{-}{n}} = \sqrt{z_{\overset{-}{n}}^2 + d^2} \tag{4-50}$$

We next note that

$$H_x(\underline{J}_n , \underline{J}'_n ; \underline{0}) = 2 H_x(\underline{J}_n ; \underline{0}) \tag{4-51}$$

$$E_y(\underline{J}_n , \underline{J}'_n ; \underline{0}) = 2 E_y(\underline{J}_n ; \underline{0}) \tag{4-52}$$

for $n = 1,2,\ldots, NB$.

Therefore, by (4-38) - (4-40), (4-42), (4-43), (4-45), (4-46), (4-48), (4-51) and (4-52), we have

$$T_{11} = T_{1,NB} = \frac{-e^{-jkd}}{2\pi d} \tag{4-53}$$

$$T_{31} = - T_{3,NB} = \frac{e^{-jkd}}{2\pi d^2} \tag{4-54}$$

$$T_{1n} = \frac{-z_n d}{2\pi} \left(\frac{1}{r_{on}^3} + \frac{jk}{r_{on}^2}\right) e^{-jkr_{on}} \tag{4-55}$$

$$T_{3n} = \frac{d}{2\pi} \left[\left(\frac{1}{r_{\overset{+}{n}}^3} + \frac{jk}{r_{\overset{+}{n}}^2}\right) e^{-jkr_{\overset{+}{n}}} - \left(\frac{1}{r_{\overset{-}{n}}^3} + \frac{jk}{r_{\overset{-}{n}}^2}\right) e^{-jkr_{\overset{-}{n}}} \right] , \tag{4-56}$$

for $n = 2,3,\ldots, NB-1$

$$T_{2n} = 0 , \quad \text{for} \quad n=1,2,\ldots, NB \tag{4-57}$$

Finally, we note that for the chosen incident field, substituion of $I_2^i = 0$ in (4-35) and $T_{2n} = 0$ in (4-57) into (2-1) gives $V_2 = 0$. This means that there is no equivalent dipole in the $\underline{t}_2$ direction for the incident field specified in Fig. 3.1. In order to save computation, we expand $\underline{M}$ as a two-term expansion with $\underline{M}_1$ representing $\underline{K\ell} = \underline{t}_1$ and $\underline{M}_2$ representing $\underline{I\ell} = -j\omega\epsilon\ \underline{n}$, and remove the corresponding columns or rows due to $\underline{K\ell} = \underline{t}_2$ in the matrices $\vec{I}^i$, $[Y^a + Y^b]$, $[T]$, $[\tilde{T}]$ and $\vec{V}$.

V. EQUIVALENT CIRCUIT

In this section, an equivalent circuit for the transmission line mode on the wire at $z = 0$ is derived. Basically, the approach is similar to that by Yang and Harrington [11] who dealt with the problem of a narrow slot backed by an infinitely-long wire. Actually, the network they derived is good only for an aperture-wire problem for which $I_1 = I_{NB}$. In the following we generalize the network such that it can be applied to any aperture-wire problem. It is known that an equivalent circuit of a wire passing by an arbitrary aperture is a two port network as shown in Fig. 5.1. The parameters $[Z^e]$ depend only on the geometry of the problem. They can be obtained by removing the incident fields from the system and applying mathematically arbitrary excitations to the transmission line formed by the wire and the aperture-perforated conducting plane. The sources V_1^e and V_2^e can be obtained from the incident fields and $[Z^e]$.

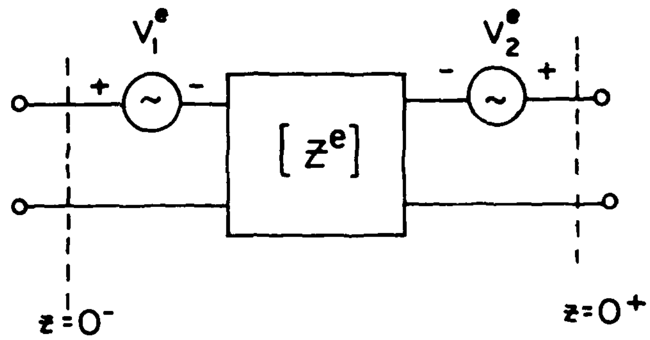


Fig. 5.1. Equivalent circuit.

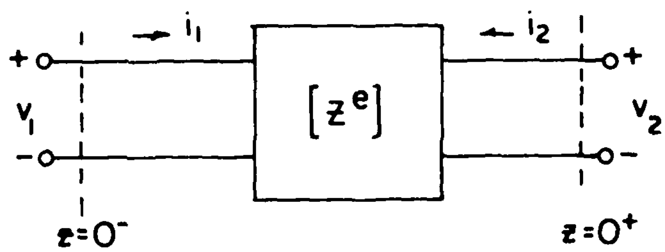


Fig. 5.2. Reference directions for i and v.

(A) Network Parameters $[Z^e]$

To obtain the elements of $[Z^e]$, we use the definitions

$$v_1 = Z_{11}^e \, i_1 + Z_{12}^e \, i_2 \tag{5-1}$$

$$v_2 = Z_{21}^e \, i_1 + Z_{22}^e \, i_2 \tag{5-2}$$

where i_1, i_2, v_1 and v_2 are port currents and voltages at ports 1 ($z=0^-$) and 2 ($z=0^+$), respectively, with reference directions shown in Fig. 5.2.

To represent an excitation, we mathematically apply a TEM current wave $I^+ e^{-jkz} \underline{u}_z$ to the transmission line. The corresponding incident fields from region b of Fig. 2.1 are then

$$\underline{H}^{ib} = I^+ \underline{h} \, e^{-jkz} \tag{5-3}$$

$$\underline{E}^{ib} = V^+ \underline{e} \, e^{-jkz} \tag{5-4}$$

where I^+ and V^+ are the mode current and voltage, and $\underline{e}$ and $\underline{h}$ are transverse electric and magnetic fields of TEM mode. We now have an incident wave $I^+ e^{-jkz} \underline{u}_z$, and two outward traveling currents $\hat{I}_1 e^{jkz} \underline{u}_z$ and $\hat{I}_{NB} e^{-jkz} \underline{u}_z$ excited by the aperture, as shown in Fig. 5.3. $\hat{I}_1$ and $\hat{I}_{NB}$ can be obtained by solving (2-1) and (2-2) with the incident fields defined in (5-3) and (5-4).

Combining Fig. 5.2 and Fig. 5.3, we obtain the port currents i_1 and i_2, and port voltages v_1 and v_2 at ports 1 and 2, respectively. They are

$$i_1 = I^+ + \hat{I}_1 \tag{5-5}$$

$$i_2 = -(I^+ + \hat{I}_{NB}) \tag{5-6}$$

$$v_1 = Z_o(I^+ - \hat{I}_1) \tag{5-7}$$

$$v_2 = Z_o(I^+ + \hat{I}_{NB}) \tag{5-8}$$

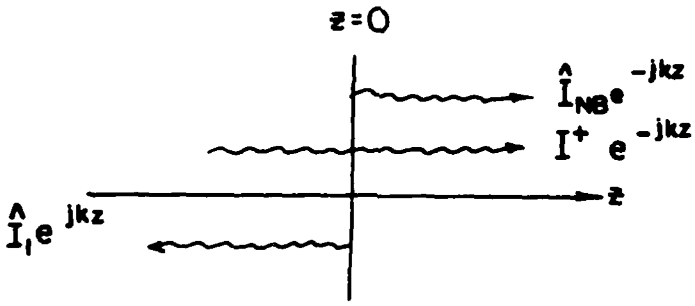


Fig. 5.3. A TEM incident current wave $I^{+}e^{-jkz}\underline{u}_z$ produces two outgoing currents $\hat{I}_1 e^{jkz}\underline{u}_z$ and $\hat{I}_{NB}e^{-jkz}\underline{u}_z$.

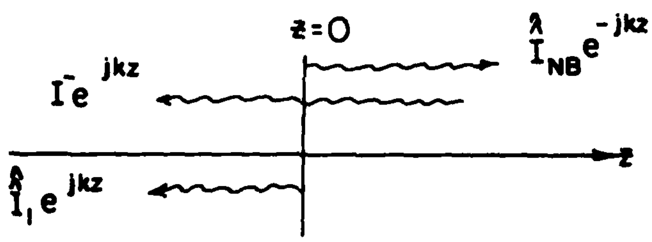


Fig. 5.4. A TEM incident current wave $I^{-}e^{jkz}\underline{u}_z$ produces two outgoing currents $\hat{I}_1 e^{jkz}\underline{u}_z$ and $\hat{I}_{NB}e^{-jkz}\underline{u}_z$.

where $Z_o = 60 \log (\frac{2d}{r_B})$ is the characteristic impedance of the transmission line. Since $[Z^e]$ is independent of the excitation, (5-1) and (5-2) hold for the port currents and voltages in (5-5) - (5-8). That is

$$\dot{v}_1 = Z^e_{11} \hat{i}_1 + Z^e_{12} \dot{i}_2 \tag{5-9}$$

$$\dot{v}_2 = Z^e_{21} \hat{i}_1 + Z^e_{22} \dot{i}_2 \tag{5-10}$$

To solve for $[Z^e]$, we need two more equations relating the port currents and voltages to $[Z^e]$ in addition to (5-9) and (5-10). One can apply a TEM current wave $I^- e^{jkz} \underline{u}_z$ to the wire. Again, the aperture excites two outgoing traveling currents $\hat{\hat{I}}_1 e^{jkz} \underline{u}_z$ and $\dot{I}_{NB} e^{-jkz} \underline{u}_z$, as shown in Fig. 5.4. The port currents and voltages for this case can be obtained by combining Fig. 5.2 and Fig. 5.4, giving

$$\dot{i}_1 = I^- + \dot{I}_1 \tag{5-11}$$

$$\dot{i}_2 = - (I^- + I_{NB}) \tag{5-12}$$

$$\hat{v}_1 = - Z_o(I^- + I_1) \tag{5-13}$$

$$\dot{v}_2 = Z_o(\hat{\hat{I}}_{NB} - I^-) \tag{5-14}$$

Similarly to (5-9) and (5-10), we have a pair of equations relating these port currents and voltages to $[Z^e]$. They are

$$\hat{v}_1 = Z^e_{11}\ \hat{i}_1 + Z^e_{12}\ \hat{\hat{i}}_2 \tag{5-15}$$

$$\hat{v}_2 = Z^e_{21}\ \hat{\hat{i}}_1 + Z^e_{22}\ \hat{i}_2 \tag{5-16}$$

Thus, by solving (5-9), (5-10), (5-15), and (5-16) with port currents and voltages defined in (5-5) - (5-8) and (5-11) - (5-14), we can obtain $[Z^e]$.

However, we note that if

$$I^- = - I^+ \tag{5-17}$$

then

$$\hat{I}_1 = - I_{NB} \tag{5-18}$$

$$I_{NB} = - \dot{I}_1 \tag{5-19}$$

This can be verified by introducing a new coordinate system where the z coordinate is replaced by -z in Fig. 5.4. Thus, we have $I^- e^{-jkz} (-\underline{u}_z)$ producing $\hat{I}_1 e^{-jkz} (-\underline{u}_z)$ and $\dot{I}_{NB} e^{jkz} (-\underline{u}_z)$. Comparing these currents in the new coordinate system with those in Fig. 5.3, we obtain (5-17) - (5-19). Substituting (5-17) - (5-19) into (5-11) - (5-14), we obtain

$$\dot{i}_1 = \hat{i}_2 \tag{5-20}$$

$$\hat{i}_2 = \dot{i}_1 \tag{5-21}$$

$$\dot{v}_1 = \dot{v}_2 \tag{5-22}$$

$$\hat{\hat{v}}_2 = \dot{v}_1 \tag{5-23}$$

Substituting (5-20) - (5-23) into (5-15) and (5-16), we obtain

$$\hat{v}_2 = Z^e_{11}\ \hat{i}_2 + Z^e_{12}\ \hat{i}_1 \tag{5-24}$$

$$\hat{v}_1 = Z^e_{21}\ \hat{i}_2 + Z^e_{22}\ \hat{i}_1 \tag{5-25}$$

By (5-9), (5-10), (5-24) and (5-25) we have

$$Z^e_{11} = Z^e_{22} = \frac{\hat{v}_1\hat{i}_1 - \hat{v}_2\hat{i}_2}{\hat{i}_1^2 - \hat{i}_2^2} \tag{5-26}$$

$$Z^e_{12} = Z^e_{21} = \frac{\hat{v}_2\hat{i}_1 - \hat{v}_1\hat{i}_2}{\hat{i}_1^2 - \hat{i}_2^2} \tag{5-27}$$

We therefore obtain the network elements by simply applying a TEM current wave $I^+ e^{-jkz}\ \underline{u}_z$ and solving for the outward traveling currents on the wire.

Furthermore, (5-26) and (5-27) show that the network is symmetric ($Z^e_{11} = Z^e_{22}$) and reciprocal ($Z^e_{12} = Z^e_{21}$), and therefore it is equivalent to a symmetric Tee network as shown in Fig. 5.5. The network elements are

$$Z^e_1 = Z^e_{11} - Z^e_{12}$$

$$= \frac{-(\hat{I}_1 + \hat{I}_{NB})\ Z_0}{2I^+ + \hat{I}_1 + \hat{I}_{NB}} \tag{5-28}$$

$$Z^e_2 = Z^e_{12}$$

$$= \frac{2I^+(I^+ + \hat{I}_{NB})\ Z_0}{(\hat{I}_1 - \hat{I}_{NB})(2I^+ + \hat{I}_1 + \hat{I}_{NB})} \tag{5-29}$$

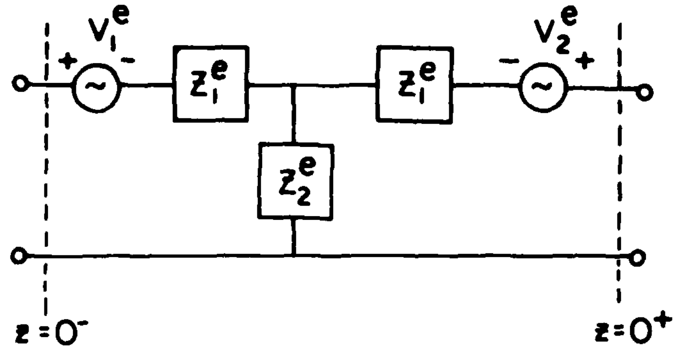


Fig. 5.5. Equivalent network for the general problem in Fig. 2.1.

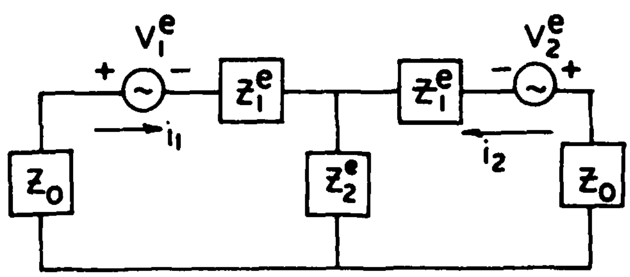


Fig. 5.6. Equivalent circuit for the calculation of V_1^e and V_2^e.

(B) Sources V_1^e and V_2^e

If both ports of Fig. 5.5 are terminated with matching loads, as shown in Fig. 5.6, the problem corresponds to that of an infinitely-long wire passing by an aperture. This is our original problem. In this case, we can solve matrix equations (2-1) and (2-2) to obtain the outward traveling currents $I_1 e^{jkz} \underline{u}_z$ and $I_{NB} e^{-jkz} \underline{u}_z$. The loop currents i_1 and i_2 in Fig. 5.6 are equal to I_1 and $-I_{NB}$, respectively. Thus, Fig. 5.6 gives the sources

$$V_1^e = -(Z_0 + Z_1^e + Z_2^e)I_1 + Z_2^e I_{NB} \tag{5-30}$$

$$V_2^e = (Z_0 + Z_1^e + Z_2^e)I_{NB} - Z_2^e I_1 \tag{5-31}$$

Note that Z_1^e, Z_2^e, V_1^e and V_2^e in (5-28) - (5-31) are proportional to Z_0. We therefore can normalize them by Z_0. This completes the derivations of the equivalent network for the general problem in Fig. 2.1.

Because of the linearity of the operators in (2-1) and (2-2), $\hat{I}_1$ and $\hat{I}_{NB}$ are proportional to I^+. Thus, we can choose $I^+ = 1$ and reduce (5-28) and (5-29) to

$$Z_1^e = \frac{-(\hat{I}_1 + \hat{I}_{NB})\, Z_0}{2 + \hat{I}_1 + \hat{I}_{NB}} \tag{5-32}$$

$$Z_2^e = \frac{2\,(1 + \hat{I}_{NB})\, Z_0}{(\hat{I}_1 - \hat{I}_{NB})(2 + \hat{I}_1 + \hat{I}_{NB})} \tag{5-33}$$

In a summary, the network is a symmetric Tee network (Fig. 5.5), and the elements defined in (5-32) and (5-33) can be obtained by removing

the incident fields from the system and applying a TEM current $e^{-jkz}\,\underline{u}_z$ to the wire. Finally, we note that for the symmetric case ($I_1 = I_{NB}$, $I_1 = I_{NB}$), Z_2^e in (5-33) is infinite and the equivalent network in Fig. 5.5 can be reduced to [11, Fig. 8] with the element and source defined in [11, Eq. (99) and (100)].

For the small aperture problem of Fig. 3.1, the source vectors $\vec{I}^i$ and $\vec{V}^i$ corresponding to the excitation $I^+ e^{-jkz}\,\underline{u}_z$ mathematically applied to the wire are derived in the following way.

The transverse electric and magnetic fields of TEM mode at center $\underline{0}$ of the small aperture are [1]

$$\underline{h}(\underline{0}) = \frac{\underline{u}_x}{d} \tag{5-34}$$

$$\underline{e}(\underline{0}) = \underline{h}(\underline{0}) \times \underline{u}_z \tag{5-35}$$

By (4-22) - (4-24) with $\underline{H}^{ia} = \underline{E}^{ia} = \underline{0}$ and (5-3), (5-4), (5-34) and (5-35), we have

$$\vec{V}^i = \vec{0} \tag{5-36}$$

$$\vec{I}^i = \begin{bmatrix} \frac{1}{d} \\ 0 \\ \frac{-j\,Z_0}{d} \end{bmatrix} \tag{5-37}$$

In (5-37), $V^+ = Z_0 I^+$ and $I^+ = 1$ are used. Note that $I_2^i = 0$ in (5-37) and $T_{2n} = 0$ in (4-57). Hence, as stated before, we can reduce $\underline{M}$ to a two term expansion and remove the corresponding rows or columns in $[Y^a + Y^b]$, $[T]$, $[\tilde{T}]$, $\vec{I}^i$ and $\vec{V}$.

VI. NUMERICAL RESULTS AND DISCUSSION

A computer program was developed to calculate the current distribution on an infinitely-long wire, the current elements of the equivalent dipoles for an arbitrarily-shaped small aperture whose polarizabilities are given, and an equivalent network for the transmission line mode. Fortunately, analytical expressions of polarizabilities for circular aperture [7] and elliptical apertures [9] exist, and numerical solutions for any other shaped apertures have been developed [12], [13], [14]. In this section, we present the computational results for the problem of a small circular aperture of radius r_A backed by a thin infinitely-long wire of radius r_B at $y = d$.

In Fig. 6.1, we show the distribution of the total and the evanescent currents on the wire, for the case of $r_A = r_B = 0.001\lambda$ and $d = 0.1\lambda$. The real and the imaginary parts of total current are shown in Fig. 6.1(a) and (b), respectively. The real and the imaginary parts of evanescent current are shown in Fig. 6.1(c) and (d), respectively. It is seen that the evanescent current rapidly drops to zero beyond a small region near the aperture. Furthermore, both the outward traveling current and the evanescent current are discontinuous at the location of the aperture ($z = 0$), but the total current is continuous everywhere along the wire. This shows that the number NB of the current expansions on the wire must be even, and the continuity of the total current is assured by replacing the point charge $\delta(z)$ by a pulse function as stated before. In fact, the discontinuities of the evanescent and the outgoing traveling

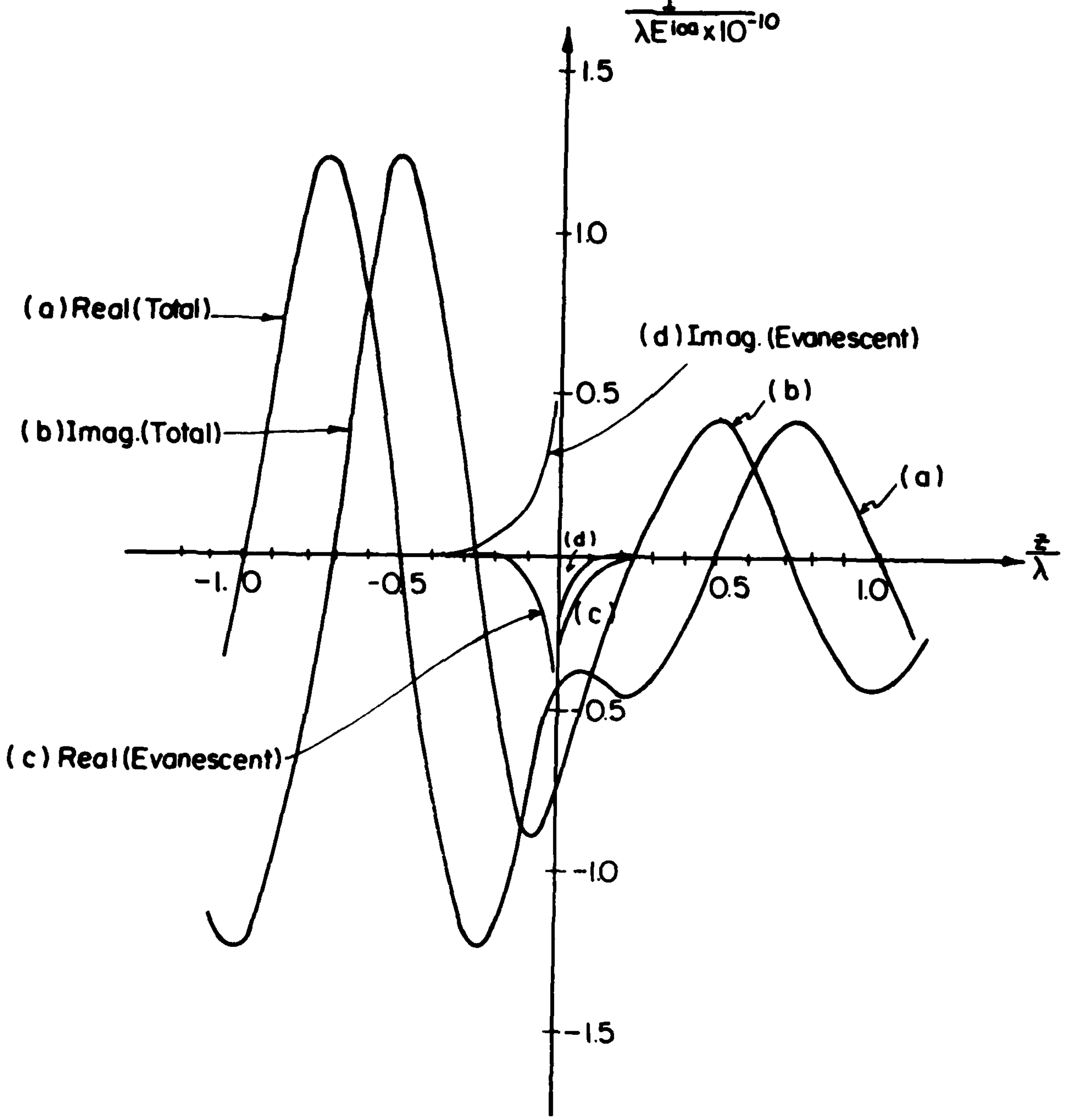


Fig. 6.1. The current distribution on the wire induced by a small circular aperture, for $r_A = r_B = 0.001\lambda$ and $d = 0.1\lambda$.
(a) The real part of total current. (b) The imaginary part of total current. (c) The real part of evanescent current. (d) The imaginary part of evanescent current.

current at $z = 0$ are expected from the results in Appendix A, since $I_1 = I_{NB}$ for magnetic dipole problem and $I_1 = - I_{NB}$ for electric dipole problem.

In Fig. 6.2, we illustrate the variations of the equivalent circuit parameters when the distance d between the wire and the aperture is changed, for the cases $r_A = 0.01\lambda$ and $r_B = 0.001\lambda$. Figure 6.2(a) shows that the imaginary part of Z_1^e approaches zero when $d \to r_A$. The real part is much smaller than the imaginary part and can be neglected when $d \geq r_A$. Figure 6.2(b) shows that the imaginary part of Z_2^e becomes larger when d is larger, and the real part can be neglected when $d \geq r_A$. Figure 6.2(c) shows that V_1^e is the complex conjugate of V_2^e (approximately).

Before we compare our results with those obtained by using Kajfez's [1], and Lee and Yang's [2] formulations, we show their equivalent networks in Fig. 6.3. In their formulations, d is the distance of the wire to the screen, R_0 the distance of the wire to the center $\underline{0}$ of the aperture and $R_0 = d$ for the problem considered in Fig. 3.1. If both terminals are loaded with Z_0, the problem is equivalent to an infinitely-long wire and the currents i_1 and i_2 at $z = 0^-$ and 0^+ in Fig. 6.3 correspond to I_1 and $-I_{NB}$, respectively. We therefore can obtain i_1 and i_2 from their circuits and compare the results with our I_1 and $-I_{NB}$. They use the Bethe-hole theory [7] to approximate the equivalent dipoles of the small aperture, but we have additional terms to account for the radiation neglected in the Bethe-hole theory. It is interesting to compare the current elements of equivalent dipoles using both formulas. By Bethe-hole theory, the current elements of equivalent dipoles in region b of Fig. 3.1 ($y > 0$) are

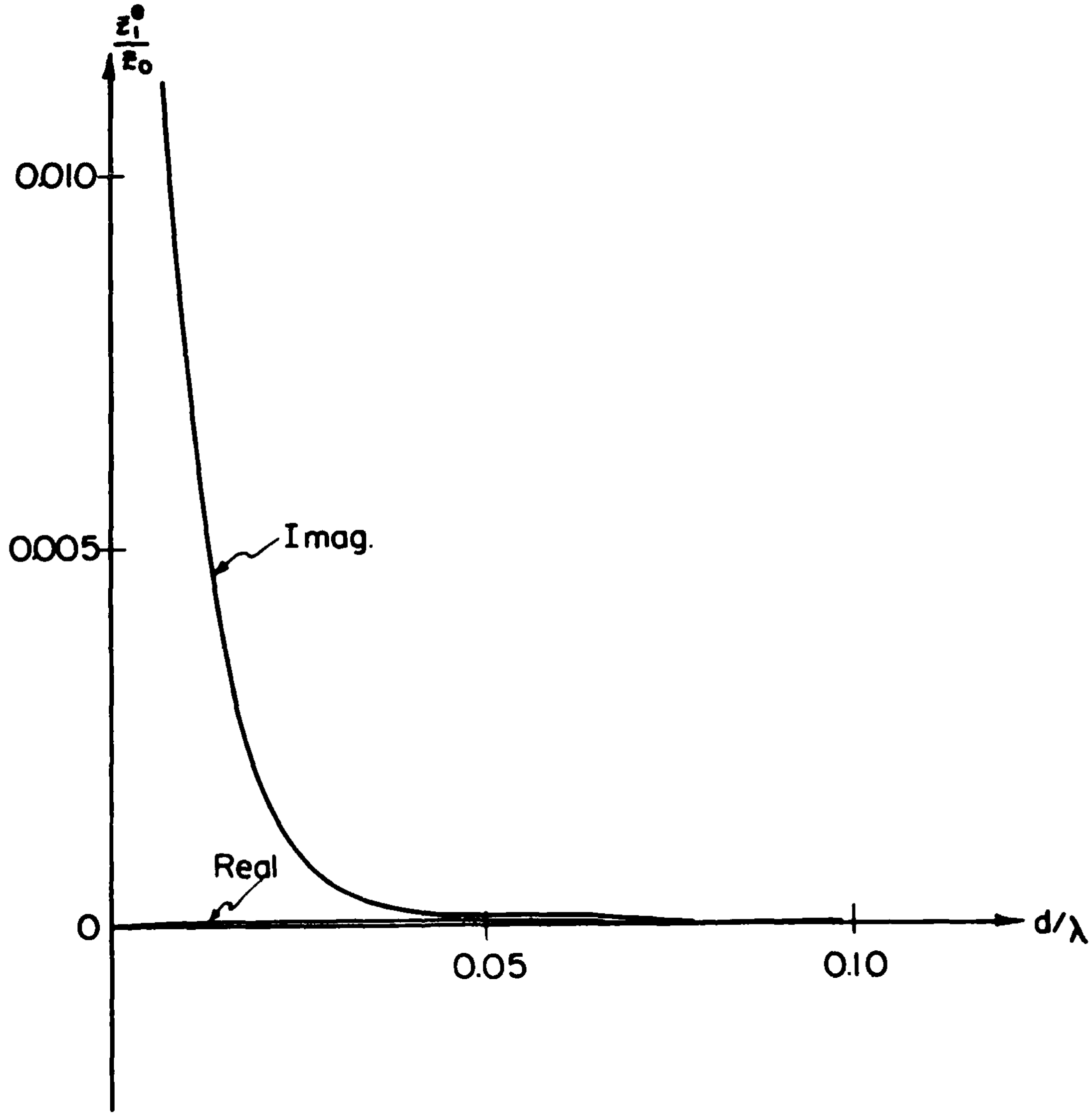


Fig. 6.2(a). The equivalent impedance Z_1^e for $r_A = 0.01\lambda$ and $r_B = 0.001\lambda$. (= 1 meter, E^{ioa} = 1 volt/meter).

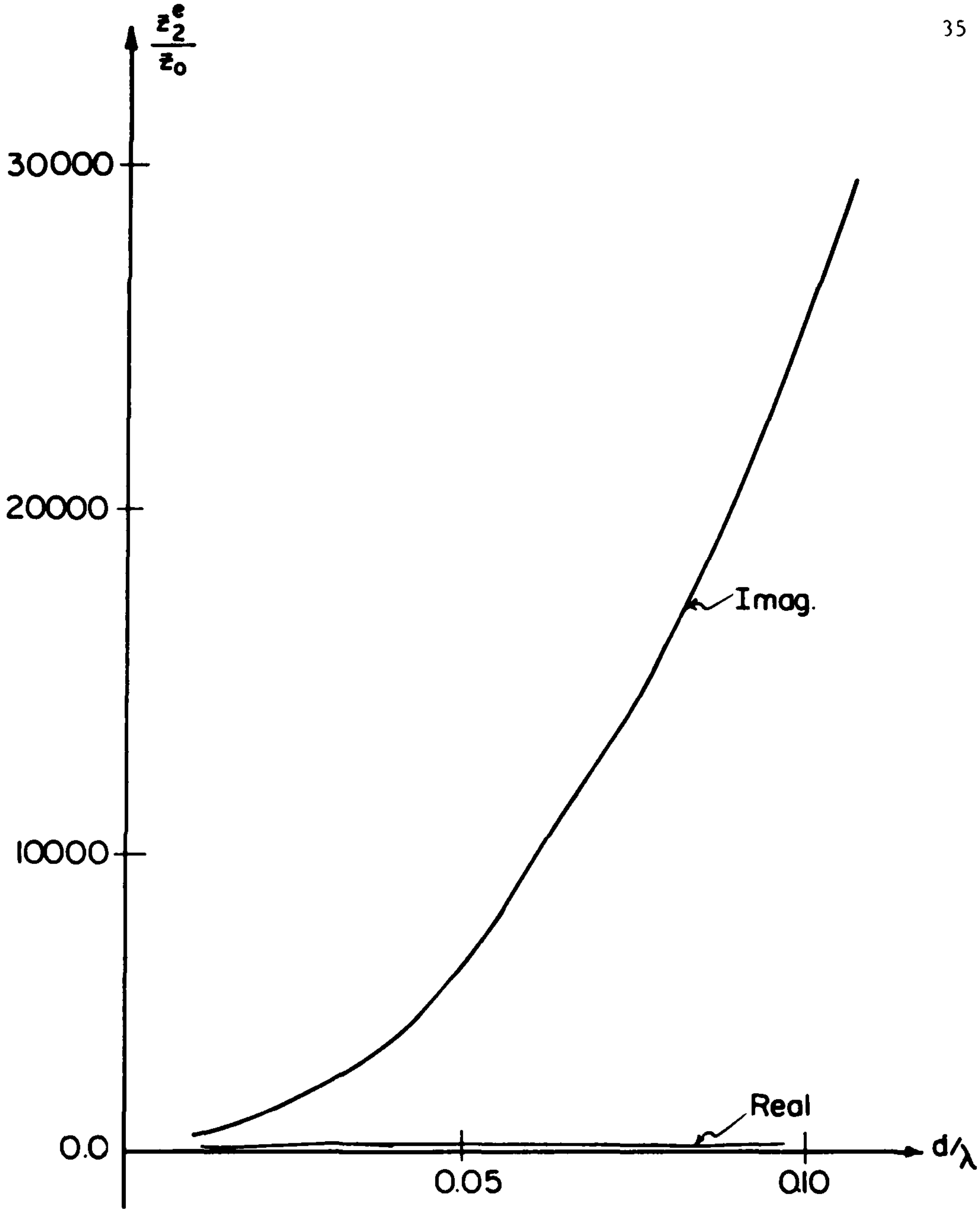


Fig. 6.2(b). The equivalent impedance Z_2^e for $r_A = 0.01\lambda$ and $r_B = 0.001\lambda$. (λ = 1 meter, E^{ioa} = 1 volt/meter).

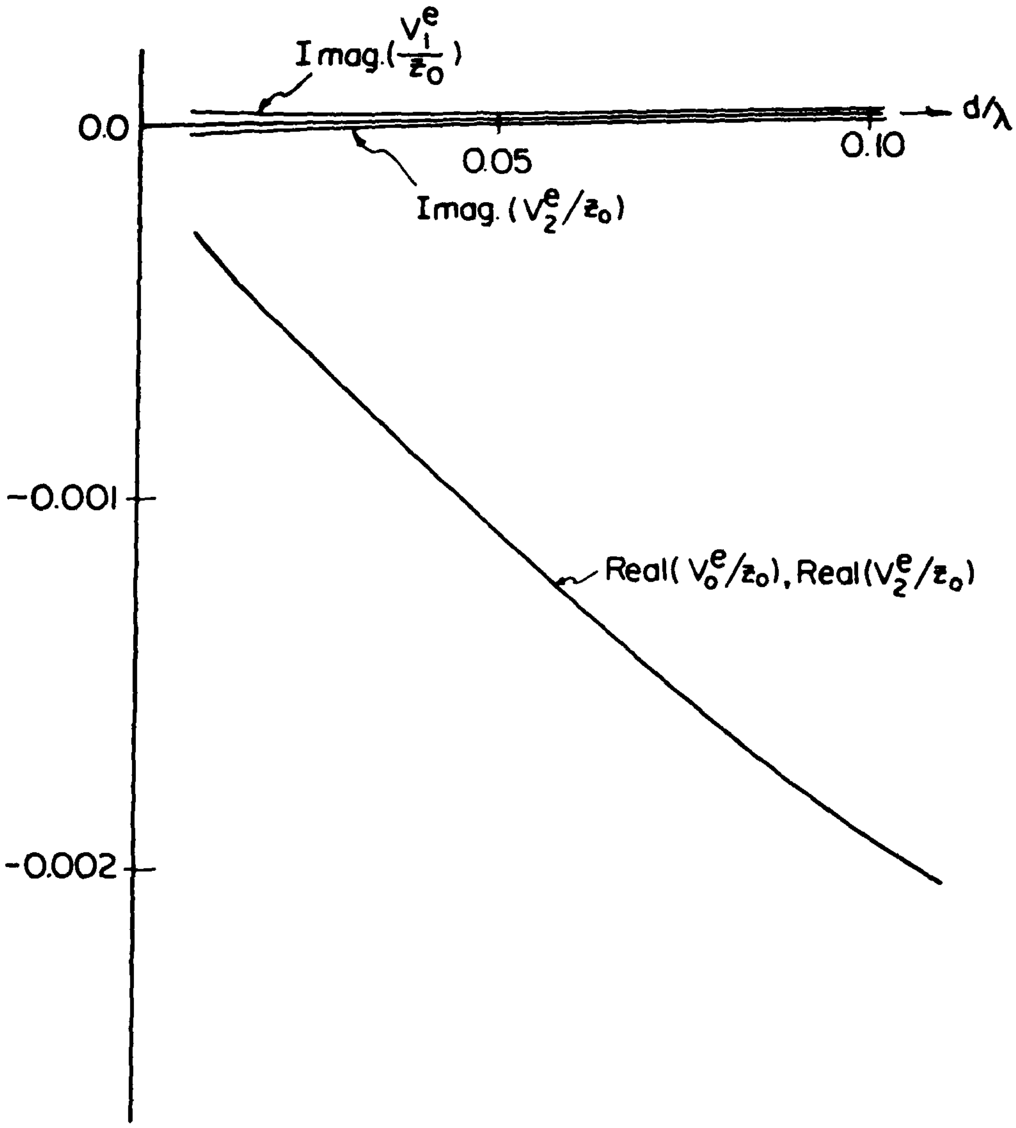
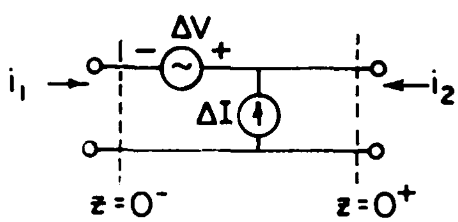
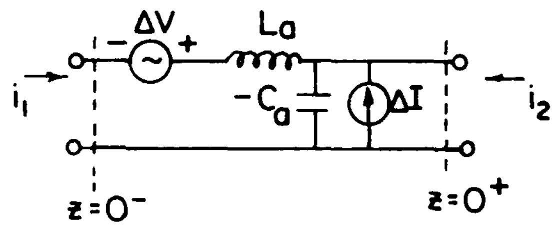


Fig. 6.2.(c) The equivalent sources V_1^e and V_2^e for $r_A = 0.01$ and $r_B = 0.001$. (= 1 meter, E^{inc} = 1 volt/meter).



(a)



(b)

$$\Delta V = j \left(\frac{d}{R_o^2}\right) {}_{mx} H_x^{ia}(\underline{0})$$

$$\Delta I = j \left(\frac{d}{R_o^2}\right) \frac{e}{Z_o} E_y^{ia}(\underline{0})$$

$$L_a = {}_{mx} \left(\frac{d}{R_o^2}\right)$$

$$-C_a = \frac{e}{Z_o^2} \left(\frac{d}{R_o^2}\right)$$

Fig. 6.3. (a) Kajfez's equivalent circuit [1].

(b) Lee and Yang's equivalent circuit [2].

$$K\ell = -j\omega\mu\alpha_{mx} H_x^{ia}(\underline{0}) \tag{6-1}$$

$$I\ell = j\omega\epsilon\alpha_e E_y^{ia}(\underline{0}) \tag{6-2}$$

The corresponding current elements of our solution are

$$K\ell = V_1 \tag{6-3}$$

$$I\ell = -j\,V_2 \tag{6-4}$$

Here V_1 and V_2 are the coefficients for $\underline{M}$ and $\underline{M}$ is reduced to a two-term expansion. Comparing (6-1) and (6-2) with (6-3) and (6-4), we can observe that the equivalent dipoles by the Bethe-hole theory depend only on the incident fields and the dimensions of the aperture, but the equivalent dipoles by our theory depend on the coupling between the wire and the aperture ([T], [T]), the geometries of the wire ([Z]) and the aperture ([Y]), and incident fields ($\vec{I}^i$).

We next compare our results with those obtained by using other methods. In Table 6.1, we compare our results of amplitudes of outward traveling currents on the wire with those obtained by using Kajfez's, and Lee and Yang's formulas, for the cases $r_A = r_B = 0.001\lambda$ and $r_A = 0.01\lambda$, $r_B = 0.001\lambda$. It is seen that our results agree closely with the other two results for the cases of $d > r_A$, but disagree with the other two results for the cases of $d \leq r_A$. Since all three methods treat a small aperture as dipoles, for the cases of $d \leq r_A$ such approximations may not be accurate. Therefore, a different method should be used for these cases.

Table 6.2 shows comparisons between our results of current elements of the equivalent dipoles and those obtained by using the Bethe-hole

Table 6.1. The amplitudes of outgoing currents, I_1 and I_{NB}, for $r_B = 0.001\lambda$. N is a constant to be divided by I_1 or I_{NB}.

r_A/λ	N	d/λ	WL/λ	NB	Our Results	Kajfez's Results	Lee & Yang's Results
					$I_1/(\lambda E^{ioa} N)$		
0.001	10^{-9}	0.10	1.5	62	-0.0027 - j0.1255	- j0.1258	- j0.1258
		0.05	1.0	62	-0.0055 - j0.2896	- j0.2896	- j0.2896
		0.01	0.5	102	-0.0005 - j2.2250	- j2.2254	- j2.2254
		0.005	0.1	62	-0.0019 - j5.7886	- j5.7906	- j5.7906
0.01	10^{-6}	0.10	2.0	82	-0.0054 - j0.1260	- j0.1258	- j0.1258
		0.05	1.0	102	-0.0037 - j0.2890	- j0.2896	-0.0001 - j0.2895
		0.02	0.2	42	-0.0061 - j0.9005	- j0.9036	-0.0019 - j0.9036
		0.01	0.1	22	-0.0177 - j2.1862	- j2.2254	*-0.0231 - j2.2251
		0.005	0.05	52	-0.1618 - j5.0727	- j5.7906	*-0.3118 - j5.7710
					$I_{NB}/(\lambda E^{ioa} N)$		
0.001	10^{-10}	0.10	1.5	62	-0.0127 - j0.4181	- j0.4194	- j0.4194
		0.05	1.0	62	-0.0389 - j0.9698	- j0.9651	- j0.9651
		0.01	0.5	102	-0.0074 - j7.419	- j7.418	-0.0001 - j7.418
		0.005	0.1	62	-0.0130 - j19.32	- j19.30	-0.0009 - j19.30
0.01	10^{-7}	0.10	2.0	82	-0.0295 - j0.4232	- j0.4194	-0.0002 - j0.4194
		0.05	1.0	102	-0.0216 - j0.9682	- j0.9651	-0.0005 - j0.9651
		0.02	0.2	42	-0.0388 - j3.0507	- j3.0121	-0.0136 - j3.0120
		0.01	0.1	22	-0.2136 - j7.8109	- j7.4180	*-0.1650 - j7.4166
		0.005	0.05	52	-1.9720 - j26.183	- j19.302	*-2.2229 - j19.209

*Note: No correction factor is used in Lee and Yang's results.

Table 6.2. The current elements $\underline{K}$· and $\underline{I}$· of equivalent dipoles in region b ($y < 0$), for $r_B = 0.001\lambda$. N is a constant to be divided by K· or I·.

r_A/λ	N	d/λ	WL/λ	NB	$K/(\ E^{ioa}\ N)$ Our Results	Bethe's Results
0.001	10^{-7}	0.10	1.5	62	0.0000 + j0.1676	j0.1676
		0.05	1.0	62	0.0000 + j0.1676	
		0.01	0.5	102	0.0000 + j0.1676	
		0.005	0.1	62	0.0000 + j0.1676	
0.01	10^{-4}	0.10	2.0	82	0.0000 + j0.1676	j0.1676
		0.05	1.0	102	0.0001 + j0.1675	
		0.02	0.2	42	0.0003 + j0.1675	
		0.01	0.1	22	0.0015 + j0.1674	
		0.005	0.05	52	0.0077 + j0.1669	
					$I/(\ E^{ioa}\ N)$	
0.001	10^{-10}	0.10	1.5	62	-0.0000 + j0.2222	j0.2222
		0.05	1.0	62	-0.0000 + j0.2222	
		0.01	0.5	102	-0.0000 + j0.2222	
		0.005	0.1	62	-0.0000 + j0.2222	
0.01	10^{-7}	0.10	2.0	82	-0.0000 + j0.2222	j0.2222
		0.05	1.0	102	-0.0000 + j0.2221	
		0.02	0.2	42	-0.0002 + j0.2211	
		0.01	0.1	22	-0.0009 + j0.2115	
		0.005	0.05	52	-0.0021 + j0.1416	

theory, for the cases $r_A = r_B = 0.001\lambda$ and $r_A = 0.01\lambda$, $r_B = 0.001\lambda$. It is seen that our results agree (exactly) with Bethe's results for the cases when $d > r_A$, but disagree with Bethe's results and the real parts of the current elements become important when $d \lesssim r_A$. We note that these real parts correspond to the radiation from the equivalent dipoles. This can be verified by using (2-1) with $\dot{I} = \dot{0}$, (4-35), (4-36), (6-3), (6-4) and recalling that the real part of $[Y^a + Y^b]$ accounts for the radiation from the dipoles.

From the comparisons in Tables 6.1 and 6.2, we can conclude that Kajfez's, Lee and Yang's and our solutions are good for the cases of $d > r_A$. As mentioned previously, all the three solutions may not be accurate for the cases of $d \lesssim r_A$. However, we take into account both the radiation from the equivalent dipoles and the interaction of the wire on the field in the aperture. Therefore, we believe that our solution can be used as an approximation for the cases of $d \lesssim r_A$.

For the problem of a loaded wire passing by a small aperture, the voltage and current of TEM wave on the wire can be obtained by applying typical transmission line formulations to the equivalent circuit for transmission line mode. Further work on solving the total current on the loaded wire and the current elements of equivalent dipoles of the small aperture can be done using a similar approach to that developed in [15]. A general solution to a wire passing by an arbitrarily-sized aperture can be done using some of the basic formulations developed here and applying triangular patching. We expect to report the results in the near future.

APPENDIX A

DIPOLE BACKED BY AN INFINITE WIRE

We here consider applications of (2-1) and (2-2) when the magnetic current in the aperture is a given magnetic dipole $\underline{K\ell} = \underline{u}_x$ or an electric dipole $\underline{I\ell} = j\,\underline{u}_y$, which is located at the origin of the coordinate and backed by a z-directed infinite wire of radius r_B at $y = d$. For this, we have $\underline{M}$ expanded as before with $\{V_1 = 1, V_2 = 0\}$ for the magnetic dipole case or $\{V_1 = 0, V_2 = -$ [illegible]$\}$ for the electric dipole case, and $\underline{J}$ on the wire expanded as before. Then the same analysis as that used for a small aperture can be applied. Thus (2-2) with $\vec{V}^i = \vec{0}$ is used to solve for $\vec{I}$.

Figure A.1 shows that the current distribution on the wire induced by the magnetic dipole is symmetric about $z = 0$. Figure A.2 shows that the current distribution on the wire induced by the electric dipole is anti-symmetric about $z = 0$. They also show that the evanescent current exists within a small region on the wire near the dipole.

Table A.1 shows that our results agree closely with those obtained by using Kajfez's formulation [1], except for the small imaginary part for the magnetic dipole case and the small real part for the electric dipole case, which are negligible. Note that NB can be either even or odd for the magnetic dipole case because of the symmetry of its excitation about $z = 0$, and NB must be even for the electric dipole case because of the anti-symmetry of its excitation.

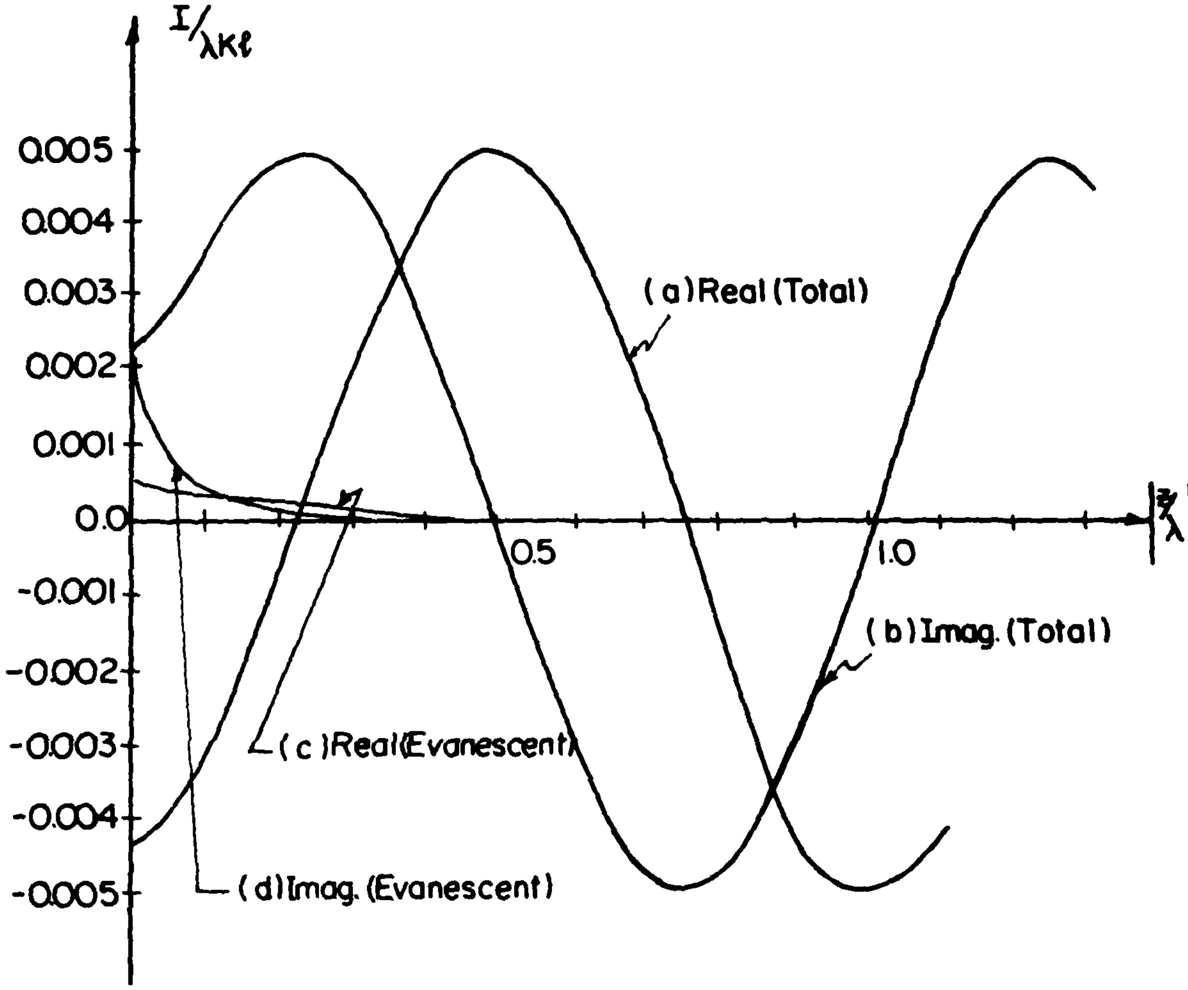


Fig. A.1. The current distribution on the wire induced by a magnetic dipole $K\ell\, \underline{u}_x$, for $r_B = 0.001\lambda$ and $d = 0.1\lambda$. (a) The real part of total current. (b) The imaginary part of total current. (c) The real part of evanescent current. (d) The imaginary of evanescent current.

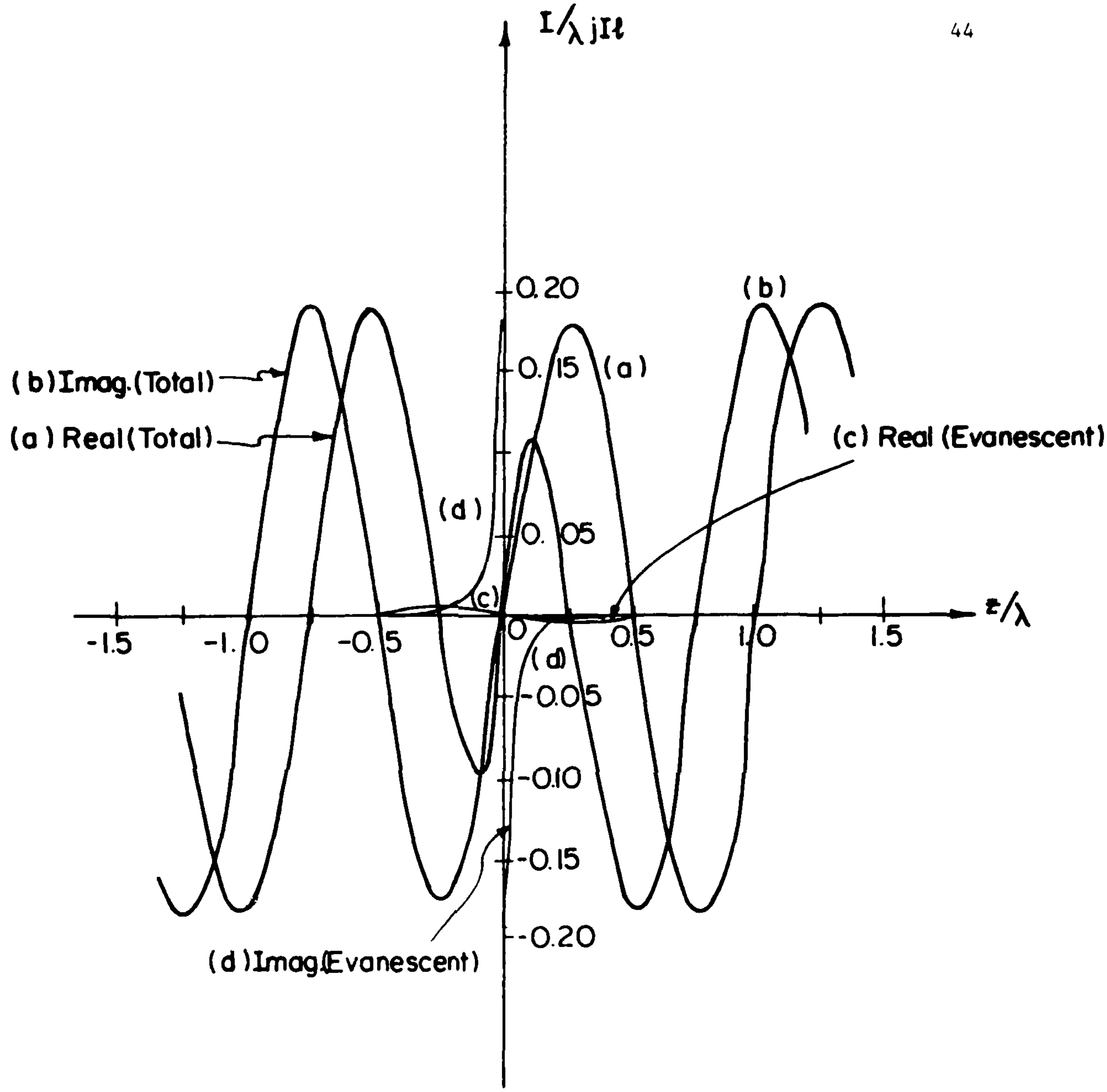


Fig. A.2. The current distribution on the wire induced by an electric dipole $I\ell\,\underline{u}_y$, for $r_B = 0.001\lambda$ and $d = 0.1\lambda$. (a) The real part of total current. (b) the imaginary part of total current. (c) The real part of evanescent current. (d) The imaginary part of evanescent current.

Table A.1. Amplitude of outward traveling current I_1 on the wire induced by a magnetic dipole $K \cdot \underline{u}_x$ or by an electric dipole $I \cdot \underline{u}_y$, for $r_B = 0.001$.

d/	$I_1/(K \cdot 10^{-2})$ induced by K			
	Our Results			Kajfez's Results
	WL/	NB		
0.20	8.0	43	-0.2131 + j0.0005	-0.2214
0.15	6.0	43	-0.3052 + j0.0000	-0.3100
0.10	4.0	43	-0.5046 + j0.0088	-0.5006
0.05	1.0	23	-1.154 + j0.0199	-1.152
0.01	0.2	43	-8.852 + j0.0006	-8.8545
	$I_1/(j I \cdot 10^{-1})$ induced by I			
0.15	6.0	42	-0.0006 - j0.1158	- j0.1169
0.10	6.0	62	-0.0005 - j0.1887	- j0.1887
0.05	2.0	42	-0.0003 - j0.4388	- j0.4343
0.01	1.0	202	-0.0007 - j3.3381	- j3.3381

APPENDIX B

COMPUTER PROGRAM LISTING AND DESCRIPTION

The computer program used to calculate the current on an infinitely-long wire, the current elements of equivalent dipoles for a small aperture of arbitrary shape, and an equivalent network for the TEM mode on the wire, is described and listed in this appendix. This program consists of subroutines IMP, CPL, SICI, MUL, MUL1, GAUSS, function P and a main program. A subroutine POL used to calculate polarizabilities of a small circular aperture is also included.

(A) Subroutine IMP

Subroutine IMP(NB, WL, RB, D, Z) computes the generalized [Z] matrix for the wire. The input variables are defined as

NB = the number of electric current expansion functions on the wire.

WL = the length (in wavelengths) where the evanescent current are considered to exist.

RB = the radius (in wavelengths) of the wire.

D = the distance (in wavelengths) from the wire to the screen.

The output is stored in a two-dimensional array Z. Minimum allocations are given by

Complex Z (NB, NB)

DO loops 10 and 20 compute the [Z] elements representing the coupling between the evanescent-current-segment and the outward traveling currents.

The elements for the couplings among the outward traveling currents are then computed. This subroutine calls function P and subroutine SICI.

Listing of IMP

```
C
C       IMP:FIND Z MATRIX:J1=CEXP(JK*Z),J(NB)=CEXP(-JK*Z),J(2)- J(NB-1):
C       PULSE EXPANSION AND IMPULSE TESTING(EXCEPT PULSE TESTING WHEN
C       CALCULATE Z(M,1) AND Z(M,NB)
C       THE 'DELTA' FUNCTION OF D(J1)/DZ , D(JNB)/DZ (POINT CHARGE
C       IS APPROXIMATED BY 'PULSE' CENTERED AT Z=0
C       WL:WIRE LENGTH FOR HIGH MODE CURRENTS(J(1) - J(NB-1))
C       NB= # EXPANSIONS
C       RB=RADIUS OF WIRE
C       D=DIST. BET. WIRE & GND PLANE
C
        SUBROUTINE IMP(NB,WL,RB,D,Z)
        COMPLEX Z(NB,NB),ZMN(5),P,JK,E
        DIMENSION DZ(5)
        COMMON/KKK/AK,PI
        COMMON/JKK/JK
        ALB=WL/FLOAT(2*(NB-2))
C
C       FIND Z(M,N), M,N=(2, NB-1)
C       M=2 , THE 1ST SUBSECTION ON WL, SO M-1 IN MO,N-1 IN NO
C       (MM,MO,MP) ,(NM,NO,NP) ARE THE POINTS ON SUBSECTION M & N
C
        D2=D*2.
        NM=0
        NO=1
        NP=2
        DO 10 M=2,NB-1
        MO=2*(M-1)-1
        MP=MO+1
        MM=MO-1
        DZ(1)=ABS(FLOAT(MO-NO)*ALB)
        DZ(2)=ABS(FLOAT(MP-NP)*ALB)
        DZ(3)=ABS(FLOAT(MP-NM)*ALB)
        DZ(4)=ABS(FLOAT(MM-NP)*ALB)
        DZ(5)=ABS(FLOAT(MM-NM)*ALB)
        DO 20 J=1,5
20      ZMN(J)=P(ALB,DZ(J),RB)-P(ALB,DZ(J),D2)
        Z(M,2)=(0.,1.)*240.*PI**2*4*ALB**2*ZMN(1)
        Z(M,2)=Z(M,2)-(0.,1.)*60*(ZMN(2)-ZMN(3)-ZMN(4)+ZMN(5))
        Z(2,M)=Z(M,2)
        DO 30 J=1,NB-1-M
        Z(M+J,2+J)=Z(M,2)
30      Z(2+J,M+J)=Z(M,2)
10      CONTINUE
```

```
C
C       FIND Z(M,1)=Z(1,M),Z(M,NB)=Z(NB,M), BY USING PULSE TESTING
C       OF J(M), M=(2,NB-1)
C
        ALS=ALB
        D22=D2**2
        L=(NB-1)/2+1
        DO 40 M=2,L
        Z0=WL*FLOAT(2*M-NB-1)/FLOAT(2*NB-4)
        DZZ=ABS(Z0)
        ZMN(1)=240.*PI*ALB*(P(ALB,DZZ,RB)-P(ALB,DZZ,D2))
        ZP=ABS(Z0+ALS)
        ZM=ABS(Z0-ALS)
        ZMN(2)=-(0,1.)*60.*(P(ALS,ZP,RB)-P(ALS,ZP,D2)
      1 -P(ALS,ZM,RB)+P(ALS,ZM,D2))
        Z(M,1)=ZMN(1)+ZMN(2)
        Z(M,NB)=ZMN(1)-ZMN(2)
        Z(1,M)=Z(M,1)
        Z(NB,M)=Z(M,NB)
        M1=NB-M+1
        Z(M1,1)=Z(M,NB)
        Z(M1,NB)=Z(M,1)
        Z(1,M1)=Z(M1,1)
        Z(NB,M1)=Z(M1,NB)
40      CONTINUE
C
C       FIND Z(1,1)=Z(NB,NB), Z(1,NB)=Z(NB,1)
C
        CALL SICI(S1,C1,RB*2*PI)
        CALL SICI(S2,C2,D2*2*PI)
        ZMN(1)=(0.,1.)*60.*(P(ALS,0.,RB)-P(ALS,0.,D2))
        ZMN(2)=60.*(-C1+C2+(0.,1.)*(S1-S2))
        Z(1,1)=-ZMN(1)
        Z(1,NB)=ZMN(1)+ZMN(2)
        Z(NB,NB)=Z(1,1)
        Z(NB,1)=Z(1,NB)
        RETURN
        END
```

(B) Subroutine SICI and Function P

Subroutine SICI (SI, CI, X) computes the sine and cosine integrals,

$$SI = \int_0^X \frac{\sin v}{v}\, dv$$

$$CI = \int_X^{\infty} \frac{\cos v}{v}$$

where X is the input, and SI, CI are the outputs. This subroutine is described in [16].

Complex function P (AL, Z, ZI) evaluates the scalar Green's function ψ defined in (4-5). The evaluation of this function is described in [10]. The input variables are

AL = ℓ_B, the half length (in wavelengths) of integration for ψ.

Z = the difference (in wavelengths) of the z-coordinate between the field point and the midpoint of source element.

ZI = ρ, the transverse coordinate (in wavelengths) of the field point.

The output is P.

Listing of SICI

```
      SUBROUTINE SICI(SI,CI,X)
      Z=ABS(X)
      IF(Z-4.)1,1,4
1     Y=(4.-Z)*(4.+Z)
      SI=X*(((((1.753141E-9*Y+1.568988E-7)*Y+1.374168E-5)
     1*Y+6.939889E-4)*Y+1.964882E-2)*Y+4.395509E-1)
      CI=((5.772156E-1+ALOG(Z))/Z-Z*(((((1.386985E-10*Y
     1+1.584996E-8)*Y+1.725752E-6)*Y+1.185999E-4)*Y+
     24.990920E-3)*Y+1.315308E-1))*Z
      RETURN
4     SI=SIN(Z)
      Y=COS(Z)
      Z=4./Z
      U=((((((((4.048069E-3*Z-2.279143E-2)*Z+5.515070E-2)
     1*Z-7.261642E-2)*Z+4.987716E-2)*Z-3.332519E-3)*Z-
     22.314617E-2)*Z-1.134958E-5)*Z+6.250011E-2)*Z+
     32.583989E-10
      V=(((((((((-5.108699E-3*Z+2.819179E-2)*Z-6.537283E-2)
     1*Z+7.902034E-2)*Z-4.400416E-2)*Z-7.945556E-3)*Z+
     22.601293E-2)*Z-3.764000E-4)*Z-3.122418E-2)*Z-
     36.646441E-7)*Z+2.500000E-1
      CI=Z*(SI*V-Y*U)
      SI=-Z*(SI*U+Y*V)+1.570796
      RETURN
      END
```

Listing of P

```
      COMPLEX FUNCTION P(AL,Z,ZL)
      COMPLEX PO,JK
      REAL I1,I2,I3,I4,LR,KL
      COMMON/KKK/AK,PI
      COMMON/JKK/JK
      R=SQRT(ZL**2+Z**2)
      PO=CEXP(-JK*R)/(4.*PI)
      IF(R .LT. (10*AL)) GO TO 30
C
C     FOR R >= 10*AL, FIND P=P(A0,A1,A2,A3,A4)
C
      KL=AK*AL
      LR=AL/R
      ZR2=(Z/R)**2
      ZR4=(Z/R)**4
      A00=-1.+3.*ZR2
      A01=3.-30.*ZR2+35.*ZR4
      A0=1.+LR**2/6.*A00+LR**4/40.*A01
      A1=LR/6.*A00+LR**3/40.*A01
      A2=-ZR2/6.-LR**2/40.*(1.-12.*ZR2+15.*ZR4)
      A3=LR/60.*(3.*ZR2-5.*ZR4)
      A4=ZR4/120.
      P=PO/R*(A0+(0.,1.)*KL*A1+KL**2*A2+(0.,1.)*KL**3*A3+KL**4*A4)
      GO TO 40
```

```
C
C       FOR R< 10*AL, FIND F=F(I1,I2,I3,I4):
C
30      Z1=Z+AL
        Z2=Z-AL
        G1=SQRT(ZL**2+Z1**2)
        G2=SQRT(ZL**2+Z2**2)
C
C       FOR Z<= AL:
C
        I1=ALOG((Z1+G1)*(-Z2+G2)/ZL**2)
        IF(Z .LE. AL) GO TO 50
C
C       FOR Z > AL:
C
        I1=ALOG((Z1+G1)/(Z2+G2))
50      I2=2.*AL
        I3=Z1/2.*G1-Z2/2.*G2+ZL**2/2.*I1
        I4=2.*AL*ZL**2+(2.*AL**3+6.*AL*Z**2)/3.
        F=PO/(2.*AL)*(I1-JK*(I2-R*I1)
     1  -AK**2/2.*(I3-2.*R*I2+R**2*I1)
     2  +(0.,1.)*AK**3/6.*(I4-3.*R*I3+3.*R**2*I2-R**3*I1))
40      RETURN
        END
```

(C) Subroutines CPL and POL

Subroutine CPL (NB, WL, D, T) computes the coupling matrix [T] of a wire to a small aperture whose magnetic current $\underline{M}$ is defined as a two-term expansion. The inputs NB, WL and D are defined in the description of IMP. The outputs are stored in a 2 × NB matrix T. The minimum allocation is given by

Complex T (2, NB)

Subroutine POL (RA, ALM, ALE) computes the magnetic polarizability α_{mx} and electric polarizability α_e for a small circular aperture of radius r_A. The input is RA = r_A, and the outputs are ALM = α_{mx} and ALE = α_e.

Listing of CPL

```
C
C        CPL: FIND THE T MATRIX FOR A SMALL  ARPERTURE
C        D:DIST. BETWEEN WIRE & ARPERTURE
C        WL:WIRE LENGTH FOR HIGHER ORDER MODE CURRENT
C        THE CENTER OF ARPERTURE IS AT (0,0,0)
C        NB:# EXPANSIONS:
C
         SUBROUTINE CPL (NB,WL,D,T)
         COMPLEX T(2,NB),CR0,CR1,CR2,CR
         COMPLEX JK
         COMMON/KKK/AK,PI
         COMMON/JKK/JK
         ALB=WL/FLOAT(2*(NB-2))
         D2=D**2
         PD=1./(2.*PI*D)
         PI2=PI*2.
         CR0=CEXP(-JK*D)
         T(1,1)=-CR0*PD
         T(2,1)=CR0/(PI2*D2)
         T(1,NB)=T(1,1)
         T(2,NB)=-T(2,1)
         DO 10 N=2,(NB-1)/2+1
C
C        THE 1ST SUBSECTION IS N=2, SO 2N-3 IN Z0:
C
         Z0=FLOAT(2*N-3)*ALB-WL/2.
         R=SQRT(D2+Z0**2)
         CR=CEXP(-JK*R)
         T(1,N)=-D*ALB/PI*(JK/R**2+1./R**3)*CR
         R1=SQRT(D2+(Z0+ALB)**2)
         R2=SQRT(D2+(Z0-ALB)**2)
         CR1=(1./R1**3+JK/R1**2)*CEXP(-JK*R1)
         CR2=(1./R2**3+JK/R2**2)*CEXP(-JK*R2)
         T(2,N)=D/PI2*(CR1-CR2)
         N1=NB-N+1
         T(1,N1)=T(1,N)
         T(2,N1)=-T(2,N)
10       CONTINUE
         RETURN
         END
```

Listing of POL

```
         SUBROUTINE POL(RA,ALM,ALE)
         ALM=4./3.*RA**3
         ALE=ALM/2.
         RETURN
         END
```

(D) Subroutines MUL, MUL1 and GAUSS

Subroutine MUL (L, M, N, A, B, C) computes the product of matrices A(L,M) and B(M,N) and stores the result in C(L,N). The inputs are integers L, M, N and matrices A, B. The output is matrix C. Minimum allocations are given by

Complex A(L,M), B(M,N), C(L,N)

Subroutine MUL1 (L, M, A, B, C) computes the product of matrix A(L,M) and vector B(M), and stored the result in vector C(L). The inputs are integers L, M, matrix A, and vector B. The output is C. Minimum allocations are given by

Complex A(L,M), B(M), C(L)

Subroutine GAUSS (N, A, B, EPS, ISW) is used to solve a linear system equation $A\,\underline{X} = B$ with complex coefficients by the method of Gaussian elimination. The inputs are the coefficient matrix A(N,N), constant vector B(N), and a small constant EPS. The output ISW is 1 if the absolute value of the pivot of column is larger than EPS, and 0 otherwise. The solution $\underline{X}$ is stored in B as an output. Minimum allocations are given by

Complex A(N,N), B(N)

Listing of MUL

```
      SUBROUTINE MUL(L,M,N,A,B,C)
      COMPLEX A(L,M),B(M,N),C(L,N),W
      DO 20 I=1,L
      DO 20 K=1,N
      W=(0,0)
      DO 10 J=1,M
10    W=A(I,J)*B(J,K)+W
20    C(I,K)=W
      RETURN
      END
```

Listing of MUL1

```
      SUBROUTINE MUL1(L,M,A,B,C)
      COMPLEX A(L,M),B(M),C(L)
      DO 10 I=1,L
      C(I)=(0.,0.)
      DO 10 J=1,M
10    C(I)=A(I,J)*B(J)+C(I)
      RETURN
      END
```

Listing of GAUSS

```
      SUBROUTINE GAUSS(N,A,B,EPS,ISW)
      COMPLEX A(N,N),B(N),C,T
      NM1=N-1
      DO 10 K=1,NM1
      C=(0.0,0.0)
      DO 2 I=K,N
      IF(CABS(A(I,K)).LE.CABS(C)) GO TO 2
      C=A(I,K)
      IO=I
    2 CONTINUE
      IF(CABS(C).GE.EPS) GO TO 3
      ISW=0
      RETURN
    3 IF(IO.EQ.K) GO TO 6
      DO 4 J=K,N
      T=A(K,J)
      A(K,J)=A(IO,J)
    4 A(IO,J)=T
      T=B(K)
      B(K)=B(IO)
      B(IO)=T
    6 KP1=K+1
      C=1./C
      B(K)=B(K)*C
      DO 10 J=KP1,N
      A(K,J)=A(K,J)*C
      DO 20 I=KP1,N
   20 A(I,J)=A(I,J)-A(I,K)*A(K,J)
   10 B(J)=B(J)-A(J,K)*B(K)
      B(N)=B(N)/A(N,N)
      DO 40 K=1,NM1
      I=N-K
      C=(0.0,0.0)
      IP1=I+1
      DO 50 J=IP1,N
   50 C=C+A(I,J)*B(J)
   40 B(I)=B(I)-C
      ISW=1
      RETURN
      END
```

(E) Main Program

The main program computes the coefficients I_n which determine the electric current on the wire, coefficients V_n which determine the current elements of the equivalent dipoles for the small aperture, and the equivalent circuit elements for the transmission line mode on the wire. It calls subroutines IMP, POL, CPL, MUL, MUL1 and GAUSS which are described previously.

The input data are assigned to the program according to the DATA statement, an example is

```
DATA NB/12/, WL/0.1/, RB/0.001/, RA/0.01/, D/0.01/
```

where NB, WL, RB, RA and D are defined previously. The solutions are stored in an output file named 'OUT' whose I/O unit is assigned to be 21 according to the OPEN statement.

In DO loop 30, current distributions on the wire and the aperture are computed twice. The first time is for TEM wave incident from region b, and the second time for plane wave incident from region a. During calculating circuit elements Z_1^e and Z_2^e, a test is made to avoid negative value of $Re(Z_1^e)$ or $Re(Z_2^e)$. If the negative value is negligible, a value of zero is assigned to it. If it is a large negative number, an error message "*** STOP, NEED LARGER WL & NB ***" appears in the output and the program stops. Usually, NB and WL must be chosen in such a way that WL/(2NB-4) is limited in the range between RB and D, and $WL/D \geq 10$. In addition, NB must be an even integer number, and the larger D is, the larger WL/D is.

Minimum allocations are given by

Complex Z(NB, NB), YZ(NA, NA), YT(NA, NB),

GZ(NB, NB), DGZ(NB, NB), VI(NB),

VM(NA), ZZ(NB, NB), YI(NA),

T(NA, NB), TT(NB, NA)

Listing of Main Program

```
      COMPLEX Z(12,12),YZ(2,2),YT(2,12),GZ(12,12),
     1 DGZ(12,12),VI(12)
      COMPLEX VM(2),ZZ(12,12),YI(2),AI1(2),AI2(2),Z1,Z2,YTI(2)
      COMPLEX E5,V1,V2
      COMPLEX JK,T(2,12),TT(12,2)
      EQUIVALENCE (GZ,ZZ)
      COMMON/KKK/AK,PI
      COMMON/JKK/JK
      DATA NB/12/,WL/0.1/,RB/0.001/,RA/0.01/,D/0.01/
      DATA ALAMDA/1.0/
C
C     NB=# OF EXPANSIONS(2 TEM, NB-2 PULSE EXPANSIONS)
C     WL:LENGTH WHERE HIGHER ORDER MODE(NON-TEM) CONSIDERED
C     RB:RADIUS OF WIRE
C     RA:RADIUS OF CIRCULAR APERTURE(USED FOR SUB. POL)
C     D:DIST. BETWEEN WIRE AND APRETURE
C     OUT:OUTPUT FILE
C
      OPEN (UNIT=21,FILE='OUT')
      PI=3.1415927
      AK=2.*PI/ALAMDA
      JK=(0.,1.)*AK
      NA=2
1     CONTINUE
      WRITE (21,100) NA,NB,RA,RB,WL,D
C
C     FIND IMPEDANCE MATRIX OF WIRE,Z(NB,NB);
C     INVERSE OF ADMITTANCE MATRIX, YZ(2,2);
C     COUPLING MATRICES T(2,NB),TT(NB,2)
C
      CALL IMP(NB,WL,RB,D,Z)
```

```
C         OBTAIN DIPOLE POLARIZABILITIES ALM,ALE
C         HERE WE CONSIDER A CIRCULAR APERTURE OF RADIUS RA
          CALL POL(RA,ALM,ALE)
          E1=8.*PI/3
          E2=120.*PI
          E3=240.*PI**2*ALM
          E4=ALE*60.
          YZ(1,1)=E1/E2-(0.,1.)/E3
          YZ(1,1)=1./YZ(1,1)
          YZ(2,2)=PI**2/45.+(0.,1.)/E4
          YZ(2,2)=1./YZ(2,2)
          YZ(1,2)=(0.,0.)
          YZ(2,1)=(0.,0.)
          CALL CPL(NB,WL,D,T)
          DO 10 M=1,NB
          DO 10 N=1,2
10        TT(M,N)=-T(N,M)
C
C         MATRIX CALCULATIONS:
C
          CALL MUL(NA,NA,NB,YZ,T,YT)
          CALL MUL(NB,NA,NB,TT,YT,GZ)
          DO 20 M=1,NB
          DO 20 N=1,NB
20        DGZ(M,N)=GZ(M,N)-Z(M,N)
C
C         K=1:TEM INCIDENT (I+ =1); K=2: PLANE WAVE INCIDENT (E =1 )
C
          ZO=60.*ALOG(2.*D/RB)
          E1=PI*D
          WRITE(21,110)
          DO 30 K=1,2
          VM(1)=1./E1
          VM(2)=-(0.,1.)*ZO/(60.*E1)
          IF(K .EQ. 1) GO TO 35
          WRITE(21,120)
          VM(1)=2./(120.*PI)
          VM(2)=(0.,-2.)/60.
35        CALL MUL1(NA,NA,YZ,VM,YI)
          CALL MUL1(NB,NA,TT,YI,VI)
          DO 40 M=1,NB
          DO 40 N=1,NB
40        ZZ(M,N)=DGZ(M,N)
          CALL GAUSS(NB,ZZ,VI,1E-11,ISW)
          IF(ISW .EQ. 1) GO TO 45
          TYPE 101
101       FORMAT('    ISW=0,STOP')
          STOP
45        WRITE(21,130)
          WRITE(21,140)(VI(M),M=1,NB)
          AI1(K)=VI(1)
          AI2(K)=VI(NB)
          CALL MUL1(NA,NB,YT,VI,YTI)
          DO 60 M=1,NA
60        VM(M)=YI(M)-YTI(M)
          WRITE(21,150)
          VM(2)=(0.,-1.)*VM(2)/60.
          WRITE(21,200)(VM(M),M=1,NA)
30        CONTINUE
```

```
C
C       FIND EQUIVALENT NETWORK:Z1,Z2,V1,V2 (NORMALIZED TO ZO )
C
        E5=2.+AI1(1)+AI2(1)
        Z1=-(AI1(1)+AI2(1))/E5
        Z2=2.*(1.+AI2(1))/E5/(AI1(1)-AI2(1))
C
C       TO AVOID ANY POSSIBLE SMALL ERROR(SMALL NEGATIVE RESISTANT)
C       DUE TO THE VERY SMALL ROUNDOFF ERROR IN REAL(I(1)),REAL(I(NB))
C
        RZ=REAL(Z1)/CABS(Z1)
        IF( RZ .GT. 0.) GO TO 25
        IF( ABS(RZ) .GT. 1.E-02 ) GO TO 27
        Z1=(0.,1.)*AIMAG(Z1)
25      RZ=REAL(Z2)/CABS(Z2)
        IF ( RZ .GT. 0. )GO TO 37
        IF(ABS(RZ) .GT. 1.E-02) GO TO 27
        Z2=(0.,1.)*AIMAG(Z2)
37      CONTINUE
        E5=1.+Z1+Z2
        V1=-E5*AI1(2)+Z2*AI2(2)
        V2=E5*AI2(2)-Z2*AI1(2)
        WRITE(21,170) Z1,Z2,V1,V2
        ZL=ZO
        WRITE(21,190) ZL
        WRITE(21,160)AI1(2),AI2(2)
        GO TO 17
27      WRITE(21,210)
100     FORMAT(//,2X,'NA=',I4,1X,'NB=',I4,1X,'RA=',F6.4,1X,
     1  'RB=',F6.4,1X,'WL=',F6.4,1X,'D=',F6.4)
110     FORMAT(/,2X,'TEM WAVE INCIDENT')
120     FORMAT(/,2X,'PLANE WAVE INCIDENT,E=1')
130     FORMAT(2X,'ELEC. CURRENT ON WIRE,1ST & LAST ARE TEM AT Z=0-,0+')
140     FORMAT(2X,2E)
150     FORMAT(/,2X,'EQUIVALENT DIPOLE   KL, IL =')
160     FORMAT(/,2X,'TEM CURRENT AT Z=0-,0+ :',/,2X,'I1=',2E,2X,'I2=',2E)
170     FORMAT(/,2X,'EQUIVALENT NETWORK(NORMALIZED TO ZO): ',/,2X,'Z1=',
     1  2E,2X,'Z2=',2E,/,2X,'V1=',2E,2X,'V2=',2E)
190     FORMAT(/,2X,'LOAD IMPEDANCES AT TWO PORTS ZL=ZO= ',F)
200     FORMAT(2(2X,2E))
210     FORMAT(2X,'*** STOP, NEED LARGER WL & NB ***')
17      CLOSE (UNIT=21,FILE='OUT')
        STOP
        END
```

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8